SETTLEMENT CONTROLLED OPTIMUM DESIGN OF SHALLOW FOOTINGS

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CERTIFICATE

This is to certify that the thesis 'Settlement Controlled Optimum Design of Shallow Footings' submitted by Sri R. Madan Mohan in partial fulfilment of the requirements for the degree of Master of Technology of the Indian Institute of Technology, Kanpur, is a record of bonafide research work carried out by him under my supervision and guidance. The work embodied in this thesis has not been submitted elsewhere for a degree.

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R. MADAN MOHAN

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LIST OF SYMBOLS

A_{sc}	=	Area of compression steel in the column
A_{st}	=	Area of tension steel in the column
α	=	A coefficient used while designing a column for biaxial moment
AST1	=	Area of steel in the column
AST2	=	Area of steel along the length of the footing
AST3	=	Area of steel along with width of the footing
A_c	=	Area of concrete in the column
b	=	Breadth of the column
B	=	Breadth of the footing
B_i	=	Breadth of ith footing
b_1	=	Width of the footing at the compression face
b_2	=	Width of the footing at the level of tension steel
β	=	A coefficient used in calculating bearing capacity of a layer soil
C_c	=	Compressive force in concrete in the column
C_{sc}	=	Compressive force in compression steel in the column
C	=	Unit cohesion
C1	=	Unit cohesion of the upper layer
C2	=	Unit cohesion of the lower
d	=	Effective depth of the footing
D	=	Over-all thickness of the footing
$\vec{D}$	=	Design vector
dc	=	Correction for depth
DC	=	Depth of column

D_f	=	Depth of embedment
e	=	Eccentricity
e_o	=	Initial void ratio
e_1	=	Final voids ratio
EX	=	Eccentricity in x direction
EY	=	Eccentricity in y direction
E_u	=	Undrained modulus of elasticity of the soil
F	=	Objective function
F.S.	=	Factor of safety against sliding
fck	=	Characteristics strength of concrete
fsc	=	Stress in compression steel in the column
fy	=	Characteristic strength of compression reinforcement
$g_j(\vec{D})$	=	Inequality constraint
H	=	Thickness of the layer
H1	=	Horizontal load on the column along X axis
H2	=	Horizontal load on the column along Y axis
k	=	Coefficient whose value is equal to 0.145
k_2	=	Coefficient whose value is equal to 0.03
L	=	Length of the footing
L_i	=	Length of ith footing
$l_j(\vec{D})$	=	Equality constraint
MUX	=	Moment coming on the column at the top about X axis
MUY	=	Moment coming on the column at the top about Y axis
M_{ux}	=	Moment coming on the footing about X axis
M_{uy}	=	Moment coming on the footing about Y axis
MUX1	=	Uniaxial moment capacity of the column about X axis
MUY1	=	Uniaxial moment capacity of the column about Y axis
	=	Ratio of length to breadth of footing

mv	=	Coefficient of volume change
MR	=	Moment of resistance of the footing against bending
n	=	Number of footing in a group
N	=	Standard penetration value
N_c^*	=	Modified bearing capacity factor used in place of N_c in layered soils
N_m	=	Modified bearing capacity factor
p	=	Area of steel in the column expressed as a percentage
P_u	=	Ultimate load on the column
P_o	=	Ultimate axial load capacity
P_{ux}	=	Ultimate load of the column under eccentricity , EX
P_{uy}	=	Ultimate load of the column under eccentricity, EY
q_c	=	Allowable shear stress
q_1	=	Maximum soil pressure along X axis
q_2	=	Minimum soil pressure along X axis
q_3	=	Maximum soil pressure along Y axis
q_4	=	Minimum soil pressure along Y axis
q_{all}	=	Allowable bearing pressure
q_{max}	=	Maximum pressure on the soil
q_{min}	=	Minimum pressure on the soil
q_{ult}	=	Net ultimate bearing capacity
q_v	=	Shear stress at a distance 'd' from the face of the column
r	=	Penalty parameter
R_D	=	Depth factor
R_W	=	Correction factor for water table
S	=	Spacing between the foundations

S.F.	=	Factor safety against overturning
S_o	=	Total settlement
T	=	Force in tension steel in the column
VAST	=	Total volume of steel used in the isolated column footing
VAST1	=	Volume of steel used in the column
VAST2	=	Volume of steel used in the footing
VCON	=	Total volume of concrete in the isolated column of the footing
VCON1	=	Volume of concrete in the column
VCON2	=	Volume of concrete in the footing
VEX	=	Volume of excavation
VFIL	=	Volume of filling
W	=	Vertical load coming on the column
W1	=	Inclined load coming on the column
w	=	Initial settlement
Z	=	Depth below the ground surface
	=	Unit weight of soil
ε	=	Accuracy desired.
	=	Penalty function
	=	Over Burden Pressure
	=	Bearing stress
	=	Poisson's ratio
	=	Allowable shear stress

SYNOPSIS

Shallow footings are the commonly occurring type of foundations for most Civil Engineering structures. The general practice of footing design is first to estimate the plan dimensions providing adequate safety against bearing capacity failure only, and then checking for settlement; if the settlement is with in the permissible value, detailed structural design are carried out on the basis of the obtained footing dimensions. Detailed investigations on the choice of proper shape and dimensions on the economy are seldom made.

As such, in the present investigation an attempt has been made to provide an easy and a direct approach for the optimal limit state design of an isolated column footing, with due attention shown towards the settlement and other essential aspects for a safe design. Plan dimensioning of a group of footings is generally done by the conventional method based either on equal bearing pressure or on an equal settlement. A method of plan dimensioning based on allowable differential settlement can be successfully applied to achieve a considerable saving in the plan area of the footing. Hence a generalised approach for the plan dimensioning of a group of footings has also been carried out.

The efficiency of numerical methods is problem oriented; so when they are applied to a new class of problems, there is a need for critical evaluation regarding their applicability. Such studies have also been undertaken. From the investigations undertaken it has been concluded that penalty function technique along with Powell's algorithm for unconstrained minimization and quadratic fit for linear minimization are quite efficient for the problems that have been solved.

Further it has been observed that the developed general methodology to obtain the optimal solution leads to a quite substantial amount of net saving over the conventional manual design. The net saving is about 10 to 40 percent depending on the type of problem.

CHAPTER - I

INTRODUCTION

1.1 GENERAL:

Most of the time inadequate attention is paid to the economic aspects in the design of an engineering system. This results in the wastage of considerable amount of scarce resources which could have been saved and utilised in a better manner. As such, it is necessary to consider several alteranative designs in order to arrive at an optimal solution. This requires the use of computers to enable the designer to speed up the tedious and monotonous repeated computational work. The computer aided design of an engineering system requires the implementation of sophisticated algorithms for the analysis to minimize the wastage of computer time.

Most of the real life problems of design and analysis are nonlinear in nature. The mathematical problems that arise in their study stretch the limits of conventional analyses and requires new methods for their successful treatment. In the use of analytic or numerical techniques for solving complex problems, the focus for discussion falls not only on the various techniques available for the analyses but also on the art of how such mathematical procedures are applied. No one procedure or series of procedures will be the panacea that solves

all problems to the last detail. While optimization theory is well established for well defined systems and models, its efficiency is problem oriented. So in the study reported here, the usefulness of such techniques in the settlement controlled optimal design of isolated column footing and plan dimensioning of a group of footings have been presented. Parametric studies have been undertaken and reported.

Design of an isolated footing involves the proportioning and the structural design of the footing. The structural design of isolated footings has received sufficient attention; but the first part of the design i.e., proportioning of the footing in contrast, has not received the same attention.

The general practice of footing design is first to estimate the plan dimensions providing adequate safety against bearing capacity failure only, and carrying out the structural design on the basis of obtained footing dimensions. Detailed investigations on the choice of proper shape and dimensions are seldom made. It is a well known fact that the basic criterion governing the design of a foundation is that the settlement must not exceed specified permissible value. This value will vary from structure to structure. Any design which does not take in to account the settlement aspect in the analysis is a suspect from the safety point of view. However, in normal practice the settlement aspects are indirectly taken care of by using an excessively high factor of safety against

bearing capacity failure, but it results in an uneconomic design.

The reported study presents an easy and direct approach of arriving at an optimal cost design of an isolated column footing satisfying the various foundation and structural design requirements.

The bearing capacity and settlement characteristics of a group of footings is different from that of a single isolated footing because of the interference amongst the adjacent footings. The phenomenon of interference is of great importance in closely built in areas where there will be interference of stress distribution in the foundation soils. It has been investigated that one of the various factors affecting the bearing capacity and settlement of a group of foundations is the spacing between the foundations. The interference of footings on sand has been observed to cause an increase in bearing capacity and decrease in settlement with the reduction in spacing, (Singh, et.al. 1973).For a safe and economic design, it is essential that the effect of interference on the stresses and displacements in the soil is not ignored.

As such, a generalised approach for the optimal plan dimensioning of a group of footings has also been included in the reported investigation..

It is a well known fact that the efficiency of numerical procedures are to a great extent problem oriented. As such, even when an established good numerical scheme is applied to new class of problems, it needs critical evaluation. In the reported investigation, sequential unconstrained minimization technique (SUMT) is used to arrive at the optimum solution. Interior penalty techniques along with the Powell's method of unconstrained minimization and quadratic fit for linear minimization has been used. Effect of starting point, penalty parameter etc. on the efficiency of the numerical algorithm when applied to the enunciated problems have been studied and reported.

1.2 BRIEF LITERATURE REVIEW:

Very few literature is available on the optimal design of footings, most of the literature pertains to the structural aspect only with complete disregard to the settlement considerations. However, there are plenty of literatures available on the various aspects of determination of stress distribution in the soil medium, bearing capacity and settlement analysis of foundations. Only those papers which are directly relevant to the present study are referred.

1.2.1 Optimal Design of Footings:

Optimum design of isolated column footing using sequential linear programming was carried out by (Bhavikatti, et.al. 1979). In this paper, the progress of optimization was

discussed and several parametric studies were carried out. It was concluded that a designer could achieve 8-10 percent economy by keeping the effective depth about 15 percent less than that obtained in the balanced section design. This paper does not consider the settlement aspects in the design.

Subbarao, et.al.(1972) have given a method for an economic plan-dimensioning of footings subjected to uniaxial moment. However, the method does not take in to care of the tilt in the footing. White, 1956 has given a graphical method to arrive at the length and breadth of a rectangular footing subject to uni-axial moment. But, both Subbarao and White have not taken in to account of the effect of depth of embedment on the required plan-domensions of a footing.

1.2.2 Bearing Capacity:

There are plenty of papers available on this aspect of foundation engineering. However for the sake of space and brevity, only those papers which are useful to the present work are presented.

Meyerhof (1953) showed that the footings subjected to eccentric loads could be safely handled by introducing a fictitious effective width, B' = B-2e of the footing instead of the actual width, 'e' being the eccentricity.

Significant contributions to the hearing capacity analyses of layered soils have been made by Button (1953), Suckl je (1954), Sivareddy and Srinivasan (1967), James et.al. (1969). Brown and Meyerhof (1969).

The studies of Sivareddy and Srinivasan (1967), James et.al. (1969), both representing the extentions of Button's work, demonstrate the fact that the case of anisotropic layered clays and the case of single clay layer with continuously variable strength can be handled with sufficient accuracy by introducing an average strength for the layer or soil zone in question.

1.2.3 Stresses and Displacements:

Very frequently, we come across the case of a rectangular area subjected to a load of linearly increasing intensity. This case was solved in a comprehensive way by Giroud (1969, 1970a, 1973), the results of his studies are set in tables and therefore convinient for practical use.

But Giroud's procedure can not be employed when the load is not distributed linearly and the loading area is not rectangular. In such cases one can proceed using Saint Venant's principle in the following way.

The actual loaded area is divided into a number of small, usually square areas, and the load applied to them is replaced by concentrated loads . The resulting stress is then

the sum of the stresses produced by these concentrated loads. To attain an accuracy more than 95 percent at a given depth, the dimensions of the areas (squares in the case considered) must be smaller than 1/3 of the depth (Feda, 1978).

Stresses and displacements in the dase of non-homogeneous soils were studied by Gibson (1967, 1973). He presented an equation for the corner settlement of a rectangular area subjected to a uniform pressure intensity.

Thenn - De Barro's (1966) handled a three layered soil by determining a single composite modulus for the entire soil. Their method can also be extended to any number of layers.

1.3 MOTIVATION AND SCOPE OF THE WORK:

It is a well known fact that settlement and tilt governs the design of a footing subjected to eccentric loads, especially in sands. Unfortunately, no work on the optimal design of eccentrically loaded footing has been done, which highlights the aspect of settlement on its design. The present work attempts to provide an easy and a direct approach for the optimal limit state design of an isolated column footing, with due attention shown towards the settlement and other essential aspects for a safe design (Chapter 3). The idea of an engineer should not only be on a safe design, but also on an economic design which fulfills the requirements of an ideal design. In this work, an attempt has been made

to achieve economy in the design, satisfying all safety aspects.

Plan dimensioning of a group of footings is generally done by the conventional method based either on an equal allowable bearing pressure or on equal settlement. The conventional method of design leads to a lot of wastage of plan area of the footing. A method of plan dimensioning based on allowable differential settlement can be successfully applied to achieve a considerable saving in the plan area of the footing. Computer aided design can alone be a resort to this kind of problems involving several trial designs.

CHAPTER - II

BASIC PRINCIPLES AND DESIGN PROCEDURES ADOPTED IN THE PROPOSED STUDY

2.1 GENERAL:

The general principles used in investigating the problems as discussed earlier are presented in this chapter. The relevant basic concepts and approach of estimating the stresses at a point in the soil media, bearing capacity and settlement analysis of the foundation and the limit state design are presented and discussed.

2.2 LIMIT STATE DESIGN:

It is generally agreed that the stress-strain behaviour of reinforced concrete (R.C.) deviates from linearly elastic behaviour appreciably even at lower loads and radically at higher loads. Still, the 'straight line' behaviour of the stress-strain relationship or the linear elastic theory is being widely used in design because of its simplicity. Extensive research has conclusively established that linear elastic theory is inadequate for a comprehensive understanding of reinforced concrete structural elements under loads progressively increasing upto failure. Consequently, ultimate strength design (U.S.D.) recognising non-linear stress-strain relation for concrete has been accepted as a means to determine the true ultimate strength of reinforced concrete sections.

All the ultimate strength design studies on reinforced concrete structures that have been carried out have essentially looked at the strength aspect only, pushing the serviceability requirement into the background. The advent of ultimate strength design and high strength materials has resulted in more flexible structures for which the serviceability considerations are as important as the strength requirement, if not more. It is observed from practice that most common cases of damage in R.C. structures are excessive deflection, cracking etc., which affects the intended utility of the structure. The safety against these damages thus demand priority for consideration in the design. This has naturally led to the limit-state design philosophy.

2.3 DEFINITION OF LIMIT STATES:

A structure, or a part of a structure is rendered unfit for use when it reaches a particular state, called a 'limit state', in which it ceases to fulfil the function, or to satisfy the condition, for which it was designed.

It is convenient to group limit states in to three major categories as follows:

(i) Ultimate limit states: Those corresponding to maximum load carrying capacity and may be caused by the rupture of critical sections of the structure or excessive plastic deformation leading to collapse.

(ii) Serviceability limit states: Those corresponding to excessive deformation, wide cracking, excessive vibrations, undesirable damage etc.

(iii) Other limit states: Those corresponding to fatigue, lightning, fire explosions etc.

2.4 SAFETY FACTOR:

The International Stadard Organisation (ISO), aiming at unification of different methods of structural calculations and ensuring the safety of structures, has recommended a semi-probabilistic limit state method. It is assumed that the statistical distribution of loads and material strengths are available. 'Characteristic values' of loads and material strengths are then selected with specified probabilities against higher load or lower strength respectively. The practice of using a global factor of safety has been replaced by a probability-based partial factor of safety format. The design loads are then computed by multiplying the characteristic loads by specified safety factors which vary with the degree of seriousness of the particular limit state being reached, probabilities of two or more loads occuring together and the reliability of the structural theories being used. Similarly, the design stresses are obtained by dividing the characteristic strengths by another partial safety factor which accounts for the difference between the strengths of controlled specimen and the material in the real structure, and any

other possible but unpredictable reduction in strengths. With these modified loads and material strengths, safety of the structure is investigated for the relevant limit states.

2.5 DESIGN OF SHORT COMPRESSION MEMBERS UNDER BIAXIAL BENDING:

The limit state design of short members subjected to biaxial bending is carried out by using the standard design procedure given by Mallick and Gupta (1980). However, it is necessary to discuss about uniaxial bending before proceeding for biaxial bending.

Referring to Fig. 2.1,

$$T = fst.\ Ast, \quad C_{sc} = fsc.\ Asc \tag{2.1}$$

$$C_c = 0.362\ .fck\ .\ b\ .\ xu \tag{2.2}$$

$$P = C_c + C_{sc} - T \tag{2.3}$$

The moment capacity, MUX1 = P.e, is given by:

$$MUX1 = P.e = C_c(0.5\ DC - 0.416\ xu) + (C_{sc}+T)\ a \tag{2.4}$$

and $$a = (DC - 2 \times cover)\ /\ 2 \tag{2.5}$$

where,

T = force in tension steel

C_c = compressive force in concrete

C_{sc} = force in compression steel

fck = characteristic strength of concrete

DC = depth of the column

P = axial load on the column

xu = depth of neutral axis

b = breadth of column

Asc = area of compression reinforcement

Ast = area of tension reinforcement.

The design of members subjected to biaxial bending is given by the equation, (I.S. 456-1978) as

$$\left(\frac{MUX}{MUX1}\right)^{\alpha} + \left(\frac{MUY}{MUY1}\right)^{\alpha} \leq 1.0 \qquad (2.6)$$

where MUX and MUY are the moment capacities about the x and y axes due to the design loads. MUX1, MUY1 are the maximum uni-axial moment capacities for an axial load Pu, bending about x and y axes respectively.

Value of α depends on Pu/Puz

where Puz = 0.45 fck . Ac + 0.75 fy. Asc (2.7)

fy = Characteristic strength of compression reinforcement.

Asc and Ast are the areas of longitudinal compression and tension reinforcements respectively.

A_c = Area of concrete in the column.

Lastly, fsc and fst are the stresses in compression and tension steel respectively.

The value of α is as follows:

α = 1.0 for Pu/Puz < 0.2

and α = 1.0 - 2.0 for 0.2 < Pu/Puz < 0.8.

The critical section of the column is the one which is in contact with the footing.

The column in the present study is short one and hence no end condition correction is necessary.

2.6 DESIGN OF THE FOOTING:

The following are the notations used in the design of the footing, (Fig. 2.2).

W1 = inclined load on the column

W1 can be resolved in to horizontal and vertical components.

H1 = horizontal load along x axis

H2 = horizontal load along y axis

EX = Eccentricity in the 'x' direction

EY = Eccentricity in the 'y' direction

D_f = depth of embedment

γ_f = partial safety factor, equal to 1.5

d = effective depth of the footing at the centre

D = overall depth of the footing at the centre.

Referring to (Fig. 2.2),

M_{ux} and M_{uy} are the moments about x and y axis respectively, which also includes the moments caused by horizontal loads H2 and H1 on them.

i.e. $$M_{ux} = MUX + H2\ (D_f - D) \qquad (2.8)$$

$$M_{uy} = MUY + H1\ (D_f - D) \qquad (2.9)$$

The other notation are:

B = breadth of the footing

q_1 = maximum soil pressure along x axis

q_2 = minimum soil pressure along x axis

q_3 = maximum soil pressure along y axis

q_4 = minimum soil pressure along Y axis,

and jd = lever arm.

Assuming the self weights of the column and the footing as 5 percent and 10 percent of the axial load coming on the column, the maximum and minimum pressure on the soil are given by

$$q_{max} = \frac{1.155\ W}{LB} + \frac{6.0 \times M_{ux}}{LB^2} + \frac{6.0 \times M_{uy}}{BL^2} \qquad (2.10)$$

and

$$q_{min} = \frac{1.155\ W}{LB} - \frac{6.0 \times M_{ux}}{LB^2} - \frac{6.0 \times M_{uy}}{BL^2} \qquad (2.11)$$

respectively.

Soil pressures, q_1 and q_2, q_3 and q_4 are given by, (Fig. 2.2).

$$q_1 = \frac{1.155\ W}{LB} + \frac{6.0 \times M_{ux}}{LB^2} \qquad (2.12a)$$

$$q_2 = \frac{1.155W}{LB} - \frac{6.0 \times M_{ux}}{LB^2} \qquad (2.12b)$$

$$q_3 = \frac{1.155W}{LB} + \frac{6.0 \times M_{ux}}{BL^2} \qquad (2.12c)$$

$$q_4 = \frac{1.155\ W}{LB} - \frac{6.0 \times M_{uy}}{BL^2} \qquad (2.12d)$$

Now the eccentricities, EX and EY are given by,

$$EX = \frac{M_{ux}}{1.05\ W} \qquad (2.13a)$$

and $$EY = \frac{M_{uy}}{1.05\ W} \qquad (2.13b)$$

Further,

$$p_1 = \frac{1.05\ W}{A} + \frac{6.0 \times M_{ux}}{LB^2} + \frac{6 \times M_{uy}}{BL^2} \qquad (2.14)$$

$$p_2 = \frac{1.05\ W}{A} - \frac{6 \times M_{ux}}{LB^2} - \frac{6\ M_{uy}}{BL^2} \qquad (2.15)$$

where, p_1 and p_2 are the pressure causing the bending moment on the footing slab and A is the area of the footing.

Maximum bending moment at CC is given by, (Fig.2.2)

$$M1 = \gamma f \left[(p_2{+}x)\ L \left(\frac{B-b}{8}\right)^2 + \left(\frac{B-b}{12}\right)^2 \cdot L.z \right] \qquad (2.16)$$

where, $$p_2{+}x = \frac{p_2(B-b) + p_1(B+b)}{2B} \qquad (2.17a)$$

and $$z = p_1 - (p_2{+}x) \qquad (2.17b)$$

The maximum bending moment about AA is given by, (Fig. 2.2),

$$M_2 = \gamma f \left[\frac{1}{8} (p_2{+}x)\ (L-b)^2\ B + \frac{1}{12} (ZZ\ .\ (L-b)^2\ B) \right] \qquad (2.18)$$

$$\text{where,} \quad p_2+x = \frac{L(p_1+p_2) + b\ (p_1-p_2)}{2L} \tag{2.19}$$

$$\text{and} \quad ZZ = p_1 - (p_2+x) \tag{2.20}$$

Further, the moment capacity of a trapezoidal section is given by

$$MR = ((k-k_2)\ b_1 + b_2\ k_2\ )\ d^2\ fck \tag{2.21}$$

where, b_1 and b_2 are the widths of the section at the compression face and at the level of tension steel respectively.

k and k_2 are the coefficients whose value are equal to 0.145 and 0.03 respectively for M_{15} concrete.

The moment capacity MR in each direction is equated to the corresponding maximum bending moments M_1 and M_2 respectively in that direction, and the effective depth 'd' is determined in both the cases by the equation,

$$d = \sqrt{\frac{\text{Maximum bending moment}}{MR}} \tag{2.22}$$

The higher value of 'd' is selected as the effective depth 'd' , which is further checked so as to ensure that it satisfies the requirements of one-way and two-way shear.

Lastly, area of steel required along the length (L) and breadth (B) of the footing are given by,

$$AST2 = \frac{1.15\ M_2}{j.d.fy} \quad \text{and} \quad AST3 = \frac{1.15 \times M_1}{j.d.fy} \tag{2.23}$$

respectively.

2.7 DESIGN CHECKS FOR SAFETY:

The following design restrictions are checked for safety, as described below.

2.7.1 Diagonal Tension:

The nominal shear stress τ_v at the section 'aaaa' Fig. 2.2, is given by

$$\tau_v = \frac{V}{b_o . d} \qquad (2.24)$$

where, $V = \gamma_f \dfrac{1.05\ W}{L.B}(LB - (b+d)^2)$ is the shear force at the section and $b_o = 4\ (b+d)$ is the peripheral distance of the critical shear zone.

The allowable shear stress (τ_a) for M_{15} concrete given by,

$$\tau_a = 0.25 \sqrt{fck} \qquad (2.25)$$

For safety against shear failure,

$$\tau_v \leq \tau_a \qquad (2.26)$$

Notations used are the same as those described in the begining is chapter.

Check for One-Way Shear:

Referring to Fig. (2.2), the shear force (S) at the cal section '00' is given by,

$$S = p.L\left(\left(\frac{B-b}{2}\right) - d\right) \qquad (2.27)$$

, b = breadth of the column,

d = effective depth of the footing,

p = average soil pressure.

Nominal shear stress (q_v) at the critical section '00' is given by

$$q_v = \frac{S}{b'd'} \quad (2.28)$$

where, b' = width of the footing at the critical section

and d' = effective depth at the critical section.

The safety against one-way shear,

$$q_v \leq q_c \quad (2.29)$$

where, q_c is the allowable shear stress as given in I.S. 456-1978.

The thickness of the footing provided at any point is also checked for adequacy.

2.7.3 Check for Bearing Stress (σ_{cbr}):

Bearing stress σ_{cbr} is calculated at the interface between the column and the footing.

The IS 456-1978 has specified the permissible bearing stress depending on the characteristic strength of concrete, supporting area of the footing and loaded area of the column. For safety the actual bearing stress should be less then the permissible value.

$$\sigma_{cbr} \leq 0.45 \; f_{ck} \sqrt{A_1/A_2} \quad (2.30)$$

with $\sqrt{A_1/A_2} \leq 2.0$ (2.31)

where, A_1 = supporting area for bearing of a footing and

A_2 = loaded area at the column base.

2.7.4 Factor of Safety Against Sliding:

Factor of safety against sliding (F.S.) along a direction is given by,

$$F.S. = \frac{\mu \times V}{\Sigma H} \tag{2.32}$$

where, V = Total weight of the isolated column footing including the backfill and the load on the column,

μ = coefficient of firction between concrete and the soil,

and ΣH = sum of the horizontal forces in that direction.

In the present case, H1 and H2 are the only horizontal forces in their respective directions.

For safety against sliding, F.S. $\geq$ 1.5.

2.7.5 Check for Overturning (S.F):

It is the ratio of restoring moment to the moment causing overturning in that direction.

$$\text{i.e., } S.F. = \frac{\text{Restoring Moment}}{\text{Over Turning Moment}} \tag{2.33a}$$

where,

Restoring moment about the length L of the footing = VxB/2 (2.33b)

Over Turning moment about the length L of the footing = H1xDf (2.33c)

Similarly, restoring moment about B of the footing $= V \times L/2$ (2.33d)

Over Turning moment about the breadth B of the footing $= H2 \times DF$ (2.33e)

Restoring moment is the moment of all the vertical forces acting on the footing about its ~~centroid~~ toe and turning moment is the moment caused due to the horizontal load plus the moment acting in that direction.

2.8 BEARING CAPACITY:

The general aspects of bearing capacity are described below:

Factors that determine the ultimate bearing capacity of horizontal footing with a central load include (1) unit weight, shear strength and deformation characteristics of the soil, (2) the size shape , depth and roughness of the footing, (3) the water table conditions and initial stresses in the foundation soil.

The general bearing capacity equation suggested by Hansen (1970)is given by,

$$q_{ult} = C\, N_c\, S_c\, d_c\, i_c + \gamma\, D_f\, N_q\, S_q\, d_q\, i_q + 0.5\gamma\, B N_\gamma\, S_\gamma\, i_\gamma d_\gamma \qquad (2.34)$$

where, C = unit cohesion

B = least lateral dimension of the footing

N_c, N_q and N_γ are the Terzaghi's bearing capacity factors

S_c, S_q and S_γ are the shape factors.

i_c, i_q and i_γ are the load inclination factors.

Lastly, d_c, d_q and d_γ are the depth factors.

Net bearing pressure is given by $q_{ult} = q_{ult}' - \gamma \times D_f$

where q_{ult}' = gross bearing pressure (2.31)

The allowable bearing pressure (q_a) is the net bearing pressure divided by an appropriate safety factor (F).

$$q_a = \frac{q_{ult}}{F} \qquad (2.35)$$

where, D_f = depth of embedment.

2.8.1 Water Table Effects:

The presence of water table around a footing reduces the effective shear strength of a grannular soil and hence, its bearing capacity is also reduced. For a fully submerged footing, the unit weights used with the N_q and N_γ terms of the bearing capacity equation is the buoyant unit weight. Since the buoyant unit weight is about one half of the moist unit weight, the bearing capacity of a submerged footing is about one half of that of the footing well above the water table. If the water table is at a distance B below the footing, it is assumed to have no effect. When the water table is at the base of the footing, the buoyant unit weight is used only with the N term.

For water table positions intermediate between the ground-surface and the base of the footing or between the

base of the footing and a distance B below the footing, the respective unit weight is adjusted by linear interpolation. Then method is available in any standard text book on foundation engineering, (Bowles, 1977).

2.8.2 Eccentric Loads:

Footings, must frequently resist an overturning moment, which results in an eccentrically applied load. Meyerhof (1953) has proposed that the dimensions B' and L' of an equivalent conccentric footing be found and used to determine the allowable bearing pressure.

The equations for B' and L' are

$$B' = B - 2\ e_B \text{ and} \tag{2.36a}$$

$$L' = L - 2\ e_L \tag{2.36b}$$

where e_B and e_L are the eccentricity in each respective direction. There total allowable load is $q_A(B'L')$.

2.8.3 Bearing Capacity in Layered Soils:

Two cases of bearing capacity analysis of layered soils are considered. They are ,

Case 1. Soft layer overlying a stiff layer, Fig. (2.3a)

Case 2. Stiff layer overlying a soft layer, Fig. (2.3a)

The bearing capacity of the foundation in both the cases is given by Brown and Meyerhof (1969), as

$$Q_o = C1.\ N_M + Q \tag{2.37}$$

where C1 = undrained shear strength of the upper layer and 'N_M' a modified bearing capacity factor which depends on, relative shear strengths of the two layers, K =C2/C1, the relative thickness of the upper layer, H/B, as well as the foundation shape.

Q_o = ultimate bearing capacity

Vesic (1970) suggested the following expression for the modified bearing capacity N_M for the situation of a soft clay layer overlying a stiff clay layer.

$$N_M = \frac{K.N_C^* (N_C^* + \beta - 1)\left[(K+1)\ N_C^{*2} + (1+K\beta)\ N_C^* + \beta - 1\right]}{\left[K(K+1)\ N_C^* + K + \beta - 1\right]\left[(N_C^* + \beta)\ N_C^* + \beta - 1\right] - (K.N_C^* + \beta - 1)(N_C^* + 1)} \qquad (2.38)$$

where $\beta = \dfrac{BL}{2(B+L)\ H}$

B = breadth of footing

L = length of footing

H = thickness of the upper layer.

For the case of stiff clay layer overlying a soft clay layer, Brown and Meyerhof (1969) suggest the following expression for N_M

$$N_M = \frac{1}{\beta} + K\ \varsigma_C\ N_C\ (\leq \varsigma_C\ N_C) \qquad (2.40)$$

where ς_C is the correction factor.

A more general analysis has been given by (Vesic, 1970) for a rectangular footing resting on an upper stronger layer having strength parameters C1 and Ø1 underlain by a lower laye

of strength parameters C2, and $\phi 2$. The expression for ultimat bearing capacity (q_o) is given by

$$q_o = \left[q_o'' + \left(\frac{1}{K}\right) C1 \ \mathrm{Cot}\ (\phi_1) \right] \exp \left[2(1+B/L)\ K \tan \phi_1 (H/B) \right]$$

$$- \frac{1}{K} \times C_1 \times \mathrm{Cot}\ \phi_1 \qquad (2.41)$$

where $K = (1 - \mathrm{Sin}^2 \phi_1) / (1 + \mathrm{Sin}^2 \phi_1)$ (2.42)

and q_o'' = bearing capacity of the footing resting on the top of layer 2, to be evaluated with strength parameters C_2 an ϕ_2 and other characteristics of the second layer.

2.8.4 Procedure for Calculating the Average Value of N, Simso (1977):

The average value of N is calculated as follows:

(a) The average value of N between the foundation level and depth of 3B/4 below the foundation is taken and is multiplied by three giving 3N1.

(b) Average value of N between depths 3B/4 and 3B/2 is taken and is multiplied by two giving 2N2.

(c) Lastly, the average value of N between 3B/2 and 2B is found, (N3).

Then,

$$N = \frac{3N_1 + 2N_2 + N_3}{6} \qquad (2.43)$$

where, N is the standard penetration number.

2.8.5 Prediction of Allowable Bearing Pressure:

Terzaghi and Peck have given in the following expression for allowable bearing capacity for 2.5 cm settlement of individual footing, differential settlement of 2 cm.

$$q_{all} = 3.5\ (N_e - 3)\ \left(\frac{B_o + 0.3}{2B_o}\right)^2 R_W\ R_d \qquad (2.44)$$

where,

q_{all} = allowable net increase in soil pressure over existing soil pressure for a settlement of 2.5 cm in t/m^2.

N_e = Standard penetration number with applicable corrections.

B_o = Width of footing (or least lateral dimension) in meters.

R_d = Depth factor = $1 + D/B_o$

$$R_d \leq 2.0 \qquad (2.45)$$

R_W = Water reduction factor

$$= 0.5\ (1 + Z_W/B_o) \qquad (2.46)$$

Z_W = Depth of water table below foundation

$$\text{At } Z_W = 0, \quad R_W = 0.5$$
$$\text{and At } Z_W = B_o, \quad R_W = 1 \qquad (2.47)$$

Meyerhof (1956) has proposed slightly different equations, but they yield for the usual size of the footings almost the same results.

The above equation is considered conservative. The net pressure may be increased by about 50 percent (Bowles, 1968).

2.9 COMPUTATION OF STRESSES IN THE SOIL MASS:

Very frequently, we come across a rectangular area subject to a load of linearly increasing intensity. This case was solved in a comprehensive way by Giroud (1969,1970a, 1973); the results of his studies are set up in tables and therefore convinient for practical use.

The other, sufficiently reliable alternative could be to divide the actual area in to a number of small, usually square areas, and the load applied to them is replaced by concentrated loads. To attain an accuracy more than 95 percent at a given depth, the dimensions of the given area (squares, in the case being considered) must be smaller than 1/3 of the depth (Taylor 1948; Florin 1959; Myslivec al, 1970, etc.).

Boussinesq's equation for a concentrated load has been employed to calculate the stress at any point below the ground-surface. The equation is given by

$$\sigma_z = K_B \frac{Q}{Z^2} \qquad (2.48)$$

where, K_B = Boussinesq's influence factor given by:

$$K_B = \frac{3}{2\pi}\left(\frac{1}{1+(r/Z)^2}\right)^{5/2} \qquad (2.49)$$

Q = vertical load (concentrated load)

r = radial distance of the point

Z = depth below the ground surface

σ_z = vertical stress in the soil.

2.10 COMPUTATION OF SETTLEMENT:

2.10.1 The Buisman - Debeer Method:

This method, in which the settlement under a given pressure is estimated, depends on the correlations between the cone penetration resistance and the compressibility of sand. Buisman proposed the following emperical equation to obtain the value of compressibility (C) from the value of the cone penetration resistance (q_c);

$$C = \frac{1.5\ q_c}{\sigma_o'} \qquad (2.50)$$

where σ_o' is the effective overburden pressure at the depth of measurement.

Settlement can be obtained by means of the equation;

$$S = \frac{H}{C} \ln \left(\frac{\sigma_o' + \Delta\sigma}{\sigma_o'}\right) \qquad (2.51)$$

where, S is the settlement of a layer of thickness H and $\Delta\sigma$ is the increment of vertical stress at the centre of the layer. If C represents the compressibility of sand over an elemental layer of thickness ΔZ, then the settlement can be expressed as

$$S = \int_0^H \frac{1}{C} \left(\ln \left(\frac{\sigma_o' + \Delta\sigma}{\sigma_o'}\right)\right) dz \qquad (2.52)$$

or approximately,

$$S = \sum_0^H 1.53 \frac{\sigma_o'}{q_c} \Delta Z \log_{10} \left(\frac{\sigma_o' + \Delta\sigma}{\sigma_o'}\right) \qquad (2.53)$$

Based on a study of case records, Meyerhof (1965) recommended that the foundation pressure producing the allowable settlement by the Buisman-Debeer's method should be increased by 50 percent. This is approximately equal to using the following equation for the constant of compressibility

$$C = 1.9 \, q_c / \sigma_o' \qquad (2.54)$$

Schmertmann's method (1970) could not be used owing to the fact that the equation given by him holds good only when the area is subject to an uniform pressure of intensity q.

2.10.2 Consolidation Settlement:

In order to estimate consolidation settlement, the value of either the coefficient of volume change or the compression index is required.

The settlement of a layer of thickness 'H' is given by

$$Sc = \int_0^H mv \, \Delta\sigma' \, dz \qquad (2.55)$$

If mv and are assumed constant with depth, then,

$$Sc = mv \, \Delta\sigma' H \qquad (2.56)$$

or $$Sc = \frac{e_o - e_1}{1 + e_1} H \qquad (2.57)$$

or, in the case of normally consolidated clay:

$$Sc = \frac{H \, C_c \log \left(\frac{\sigma_1'}{\sigma_o'}\right)}{1 + e_o} \qquad (2.58)$$

where, e_o = initial void ratio

e_1 = final void ratio

$\Delta\sigma'$ = increase in soil pressure due to the load

C_c = compression index of the soil

mv = coefficient of volume change

σ_o' = overburden pressure at any depth and

$$\sigma_1' = \sigma_o' + \Delta\sigma' \tag{2.59}$$

2.10.3 Settlement in the Case of Layered Soils:

An approximate analysis can be undertaken by a simple extrapolation proposed by Steinbrenner (1934). Steinbrenner suggested that the displacements $\Delta\rho_a$ of the point A on the surface of limited depth Z is computed from the Boussinesq's expressions for the displacements of the points A and B ($\Delta\rho A'$ and $\Delta\rho_B'$) for an infinite depth of soil (Fig. 2.3b). Since the deflection of the point B is zero, $\Delta\rho A$ is given by

$$\Delta\rho A = \Delta\rho A' - \Delta\rho B' \tag{2.60}$$

where, $\Delta\rho A'$ and $\Delta\rho_B'$ are the Boussinesq's displacements in infinite depth of soil.

A multilayered system as in Fig.2.3b is considered and the displacements $\Delta\rho_1$, $\Delta\rho_2$, $\Delta\rho_3$, etc. , for an infinite depth of soil of modulus E (i.e., as if the whole half-space had the modulus of the upper layer) is calculated. The settlement in each layer is given by

$$\delta\rho_1 = \Delta\rho_1 - \Delta\rho_2 \tag{2.61}$$

$$\delta\rho_2 = \Delta\rho_2 - \Delta\rho_3 \tag{2.62}$$

$$\delta \rho_3 = \Delta\rho_3 - \Delta\rho_4 \tag{2.63}$$

But all the settlements are based upon the use of a uniform value of $E = E1$. To accomodate variations in E with depth, we should therefore multiply the nth layer of $E1/E_n$. Hence the total settlement $\Delta\rho$ is given by

$$\Delta\rho = \frac{E_1}{E_1}\,\delta\rho_1 + \frac{E_1}{E_2}\,\delta\rho_2$$

$$+ \frac{E_1}{E_3}\,\delta\rho_3 \quad \ldots\ldots \text{ etc.} \tag{2.64}$$

For the case where 'E' increases linearly with depth, the soil can convininently be divided into finite layers, of equal thickness and ascribed an average E for each, E_1, E_2, E_3 etc. The rate of increase of E with depth is expressed in terms of E_o (modulus at the surface). Thus,

$$E = E_o \left(1 + K \frac{z}{b} \right) \tag{2.65}$$

Initial settlement for a soil can be computed by using undrained modulus 'E_u' and Poisson's ratio 'μ' in the Boussenesq's expression for displacement (w) given by,

$$w = \frac{Q\,(1+\mu)}{2\,\mu E_u} \left[\frac{Z^2}{R^3} + \frac{2(1-\mu)}{R} \right] \tag{2.66}$$

where Q = concentrated load

Z = depth at which the displacement is required

R = radial distance of the point under consideration.

2.10.4 Allowable Settlement:

Settlement is important, even though no rupture is imminent, for three reasons: appearance of the structure: utility of the structure; and damage to the structure.

If 'S_{max}' denotes the maximum settlement, 'S_{min}' denotes the minimum settlement, the differential settlement is the larger settlement minus the smaller. Differential settlement is also characterised by angular distortion δ/l, which is the differential settlement between two points divided by the horizontal distance between them.

Lambe and Whitman (1969) have given that the allowable differential settlement for an R.C.C. building frame as 0.0025-0.0041 , where 'l' is the distance between the adjacent columns that settle by different amounts, or between any two points that settle differently.

Mac Donald and Skempton (1955), made a study of 98 buildings, most of them being R.C.C. structures. They concluded that a differential settlement of about 3.75 cm on clays and 2.5 cm on sands between two isolated footings does not affect the satisfactory performance of the structure. However, while considering the interference between adjacent footings, a differential settlement of only 2 cm has been allowed between any two foundations.

2.10.5 Allowable Total Settlement:

Lambe and Whitman, (1969) have given the following allowance for the total settlement:

Settlement $\leq$ 40 mm for footings on sands

and Settlement $\leq$ 65 mm for footings on clays.

2.10.6 Influence Factor for Footings at a Depth:

Fox (1948), proposed a correction to apply to the surface deflection which depends both on the depth of embedment and poisson's ratio ..

Therefore, the corrected settlement S_e becomes,

$$S_e = S . F_3 \qquad (2.67)$$

where, F_3 is obtained by programming Fox-equations to obtain the settlement ratio vs (Df/B) and for selected (L/B) and poisson's ratios

S = surface settlement.

Expression for F_3 can not be presented here as the expression is too lengthy to present. Expression is given by Fox in his paper, (1948).

2.11 BASIC CONCEPTS OF OPTIMIZATION THEORY:

2.11.1 General:

Optimization is the act of obtaining the best result under given circumstances and satisfying all limitations and restrictions placed on it. In the analysis or design of any engineering system, the ultimate goal is to minimize the effort required or maximize the desired benefit. Since the effort required or the benefit desired in any situation can

be expressed as a function of certain decision variables, optimization can be viewed as the process of finding the conditions that gives the maximum or minimum value of the function. Again, since the maximum of a function can be found by seeking the minimum of the negative of the same function, optimization can be taken to mean minimization, without any loss of generality.

2.11.2 Terminology:

Before coming to the discussion of various techniques of optimization, it is important to define certain basic terminologies which are generally used in optimization theory.

The numerical quantities for which the values are to be chosen in producing a decision or a solution are called 'decision variables'. A vector $\vec{D}$ containing all the decision variables in a particular problem, will often be referred to as a 'decision vector'. It should be emphasized at this point that 'a decision' is simply a set of values of the decision variables, even if it is absurd or inadequate in terms of functional requirements . If a decision meets all the requirements placed on it, will be called as an 'acceptable decision'. When a decision fails to meet any of the requirements, it will be called as an 'unacceptable decision'.

The restrictions that must be satisfied in order to produce an acceptable decision or solution are collectively called as 'constraints'. A constraint that derives from the performance or behavior requirements is called a 'behaviour

constraint'.

Any decision problem which uses optimization technique will have it's objective to obtain the best decision. The function with respect to which the decision is optimized, is called the 'objective function', and is designated as F or $F(\vec{D})$.

Now a standard form of problem statement is considered. This is presented in a generalized form and the actual physical situation need to be taken into account while formulating particular problems. After making the appropriate engineering judgements and defining all the necessary functions and limitations, the optimization problem is stated as follows:

$$\text{Minimize } F(\vec{D}) \ , \ \vec{D}^T = (d_1 \ d_2 \ \ldots\ldots \ d_n)$$

$$\text{Subject to } g_j(\vec{D}) \leq 0 \quad j = 1,2, \ldots\ldots, m$$

$$l_j(\vec{D}) = 0 \quad j = 1,2,\ldots\ldots, k$$

where, g_j and l_j represent inequality and equality constraints on the decision variables, respectively.

An optimization problem in which there is no restriction on the choice of decision vector, is called an unconstrained optimization problem; otherwise, it is called a constrained optimization problem. Most of the practical application problems are constrained; but some of the most powerful and convenient methods of solving constrained problems involve the conversion of the problem to one of unconstrained minimization, and this technique has been used in this thesis.

2.11.3 The Unconstrained Minimization Technique:

The multidimensional unconstrained minimization technique that will be used in this thesis is, Powell's (1964) conjugate direction method. Detailed descriptions of this method is obtainable in any standard text on optimization (e.g., Fox 1971, Rao 1978). In this method, at each point in the function space under consideration, there is a preferred direction along which the values of the decision variables are changed systematically by a well defined scheme, which leads to the minimum. Thus starting from an initial guess one arrives at the minimum through a sequence of improved approximations, each derived from the previous approximation. The various methods follow the same principle stated above but differ only in selecting the direction along which they search for the minimum. Powell's method is a nongradient method. Powell's method has the property of quadratic convergence. Without going into mathematical details, a brief description of the algorithm is presented as follows.

2.11.4 Powell's Conjugate Direction Method:

This method is an extension of the idea of pattern move. For an intutive understanding, Fox (1971) described the method as follows: 'Given that the function has been minimized once in each of the coordinate directions and then in the associated pattern direction, discard one of the coordinate directions in favor of the pattern direction for inclusion in the next m minimizations, since this is likely to be a better direction than the discarded coordinate direction.

After the next cycle of minimizations, generate a new pattern direction and again replace one of the coordinate directions'. The flow diagram shown in Fig. (2.4) explains the algorithm. The basic method has a tendency to choose nearly dependent direction in ill conditioned problems and the method may fail to converge to the actual minimum. One simple remedy is to reset the directions to the original coordinate vectors periodically and/or whenever there is some indication that the directions are no longer productive. Powell has recommended a more effective procedure to avoid this difficulty(Fox,1971).

The search is terminated when the relative change in the function value and in all the decision variables, between two consecutive cycles of minimization, is less than the desired accuracy.

2.11.5 Quadratic Interpolation:

In this technique the function $F(\alpha)$ is approximated by a function $H(\alpha)$ which has an easily determinable minimum point. $H(\alpha)$ is expressed as,

$$H(\alpha) = a + b\alpha + c\alpha^2$$

the minimum of which occurs where

$$\frac{dH}{d\alpha} = b + 2c\alpha = 0$$

or $$\alpha^* = -\frac{b}{2c}$$

The constants a, b and c for the approximating quadratic can be determined by sampling the function at three different α values, α_1, α_2 and α_3, and solving the equations,

$$F_1 = a + b\alpha_1 + c\alpha_1^2$$

$$F_2 = a + b\alpha_2 + c\alpha_2^2$$

and $$F_3 = a + b\alpha_3 + c\alpha_3^2$$

where F_1 denotes $F(\alpha_1)$, and so on. Solving the above three equations, the values of a, b, c and $\alpha*$ are obtained as,

$$c = (F_3-F_1)\ (\alpha_2-\alpha_1) - (F_2-F_1)\ (\alpha_3-\alpha_1)\ /$$

$$(\alpha_1-\alpha_2)\ (\alpha_2-\alpha_3)\ (\alpha_3-\alpha_1)$$

$$b = (F_2 - F_1)\ /\ (\alpha_2-\alpha_1) - c\ (\alpha_2+\alpha_1)$$

$$a = F_1 - b\alpha_1 - c\alpha_1^2 \text{ and } \alpha* = -\ b/2c$$

Now choosing 0, t and 2t for α_1, α_2 and α_3, one obtains,

$$a = F_1$$

$$b = (4F_2-3F_1-F_3)\ /\ 2t$$

$$c = (F_3+F_1-2F_2)\ /\ 2t^2$$

and $$\alpha* = (4F_2-3F_1-F_3)\ t/(4F_2-2F_3-2F_1)$$

For $\alpha*$ to correspond to a minimum of $H(\alpha)$, it must satisfy

$$\frac{d^2H}{d\alpha^2}\Big|_{\alpha=\alpha^*} > 0$$

Since H is a qudratic, this requires

$$c > 0$$

or $$(F_3+F_1) > 2F_2$$

The point $\alpha*$ is considered to be a sufficiently good approximation to the minimum of $F(\alpha)$ if

$$\frac{|H(\alpha*)-F(\alpha*)|}{|F(\alpha*)|} \leq \varepsilon$$

The flow diagram shown in Fig. 2.5 explains the algorithm.

2.11.6 The Penalty Function Method:

The basic object of Penalty Function Method is to convert the original constrained problem into one of unconstrained minimization by blending the constraints into a composite function and making it possible to ignore them at the minimization stage. In this method, the numerical solutions are sought by solving a sequence of unconstrained minimization problems.

The advantage of Penalty function method lies in the fact that the powerful, well studied and reliable algorithms for the unconstrained minimization of arbitrary functions can be used in this method.

The penalty function formulations for inequality constrained problems can be divided into two categories; interior and exterior. In the interior formulation, the unconstrained minima all lie in the feasible region and converge to the solution as a special parameter is varied. In the exterior formulation, they lie in the infeasible region an converge to the solution from outside. The advantage of interior penalty function method is that, given an initial acceptable decision, it produces an improving sequence of acceptable decisions. Moreover, we approach the constraints in such a way that they become critical only near the end. This is a desirable feature in engineering decision process because, instead of taking the optimum decision, one can choose a suboptimal but less critical decision if required. After studying the relative advantages and disadgantages of the

available methods, the interior penalty function method was selected for solving the problems involved in this thesis.

In the interior penalty function method a new function (ϕ-function) is constructed by augmenting a penalty term to the objective function. The ϕ-function is defined as

$$\phi\,(\vec{D}, r_k) = F\,(\vec{D}) - r_k \sum_{j=1}^{m} \frac{1}{g_j(\vec{D})}$$

where F is to be minimized over all $\vec{D}$ satisfying

$$g_j(\vec{D}) \leq 0\,, \quad j = 1, 2, \ldots\ldots m\,.$$

The penalty parameter 'r' is made successively smaller in order to obtain the constrained minimum of F.

The flow diagram shown in Fig. 2.6 explains the algorithm.

The interior penalty function method requires a feasible starting point for the search towards the optimum point. In cases where finding a feasible starting point is difficult, the penalty function method itself can be used for finding it (Rao, 1978). But the procedure involved is time consuming.

A promising alternative approach is to use the extended penalty function proposed by Kavlie (Basudhar, 1976). In this method the infeasible starting points are acceptable to the minimization algorithm. In this approach the ϕ-function is defined as

$$\phi(\vec{D}, r_k) = \begin{cases} F(\vec{D}) - r_k \sum\limits_{j=1}^{m} \dfrac{1}{g_j(\vec{D})} & ;\ g_j(\vec{D}) \leq \varepsilon \\ F(\vec{D}) - r_k \sum\limits_{j=1}^{1} (2\varepsilon - g_j(\vec{D}))/\varepsilon^2 & ;\ g_j(\vec{D}) > \varepsilon \end{cases}$$

where m is the total number of satisfied constraints and l is the total number of violated constraints.

$$\theta = - r_k/\delta_t$$

δ_t = a constant that defines the transition between the two types of penalty terms.

The basic principles of the optimization techniques used in this thesis has been covered very briefly. The detailed description and explanation of these methods are available in all standard texts (Fox 1971, Rao 1978).

The basic principles described in this chapter will be referred to in the subsequent chapters.

2.11.7 Convergence Criterion:

The convergence is assumed to be achieved when the change in the values of objective function and the decision vector, between two consecutive cycles, is less than the desired accuracy, i.e.,

$$\left|\frac{F_q - F_{q-1}}{F_q}\right| \leq \varepsilon_1 \tag{2.69}$$

and

$$|\vec{D}_q - \vec{D}_{q-1}| \leq \varepsilon_2 \tag{2.70}$$

The same convergence criterion is used all through, wherever checking of covergence is involved.

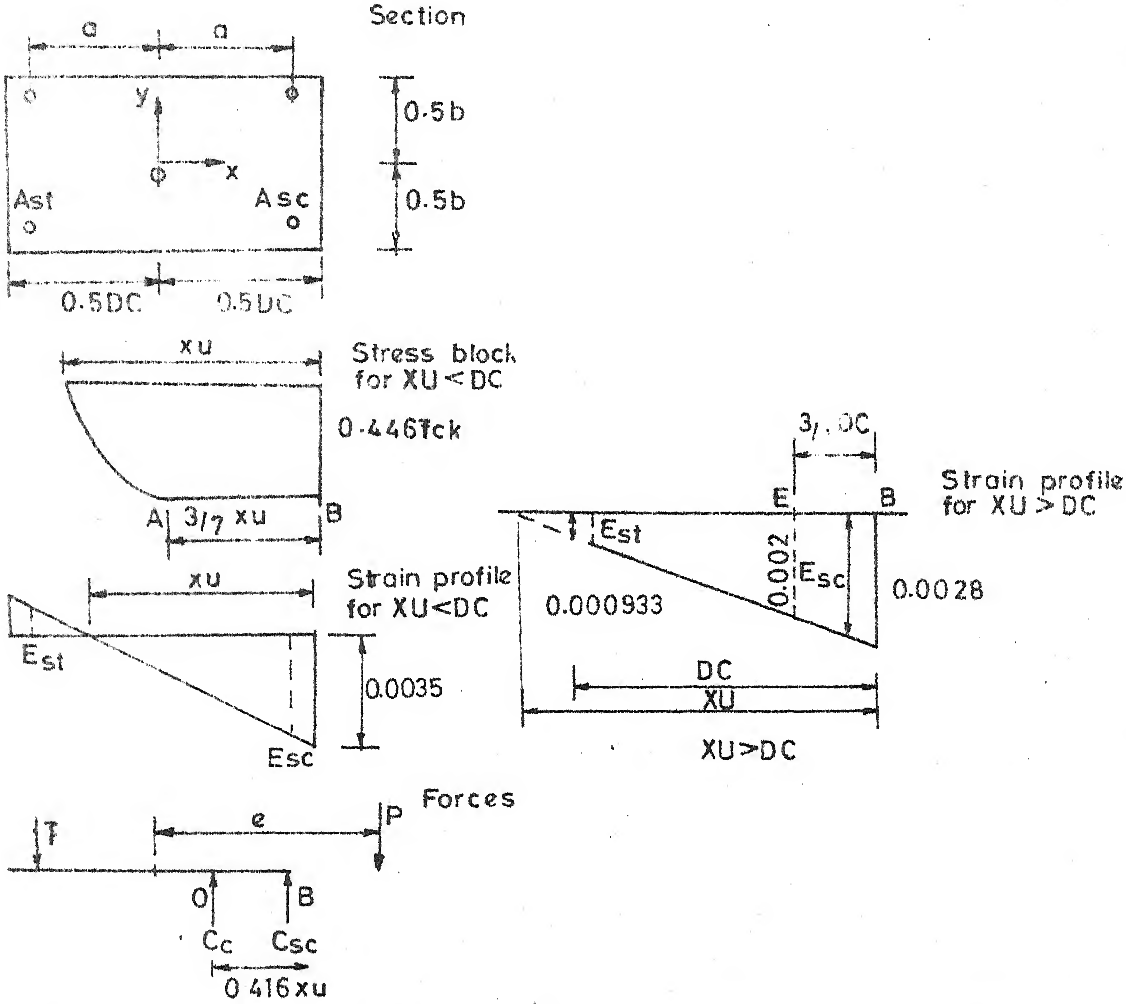


FIG. 2.1 RECTANGULAR SECTION UNDER UNIAXIAL BENDING.

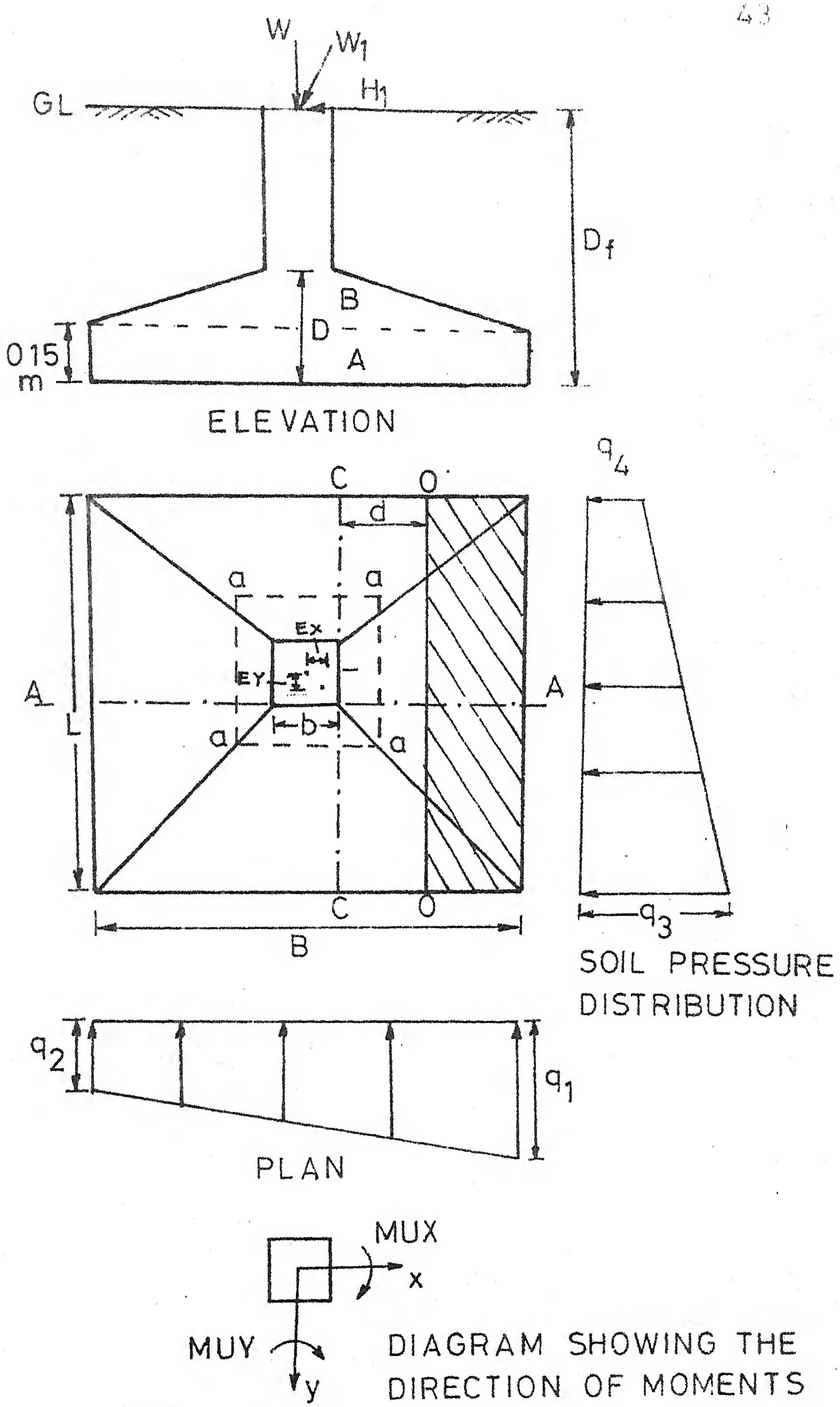
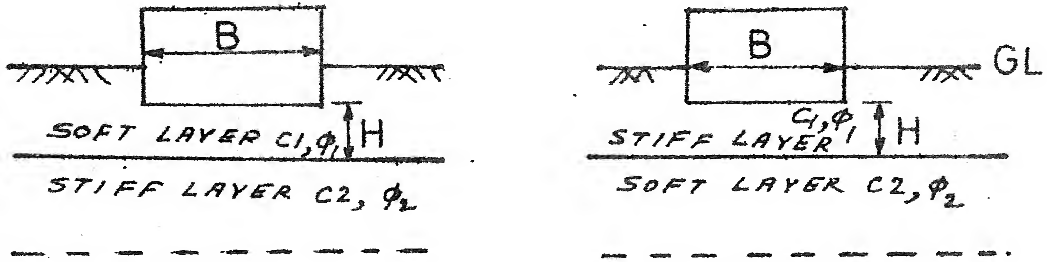
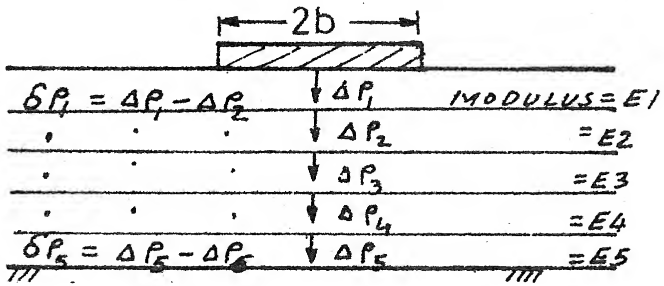


FIG. 2·2 DIAG. SHOWING THE PLAN AND ELEVATION OF THE FOOTING



(a)

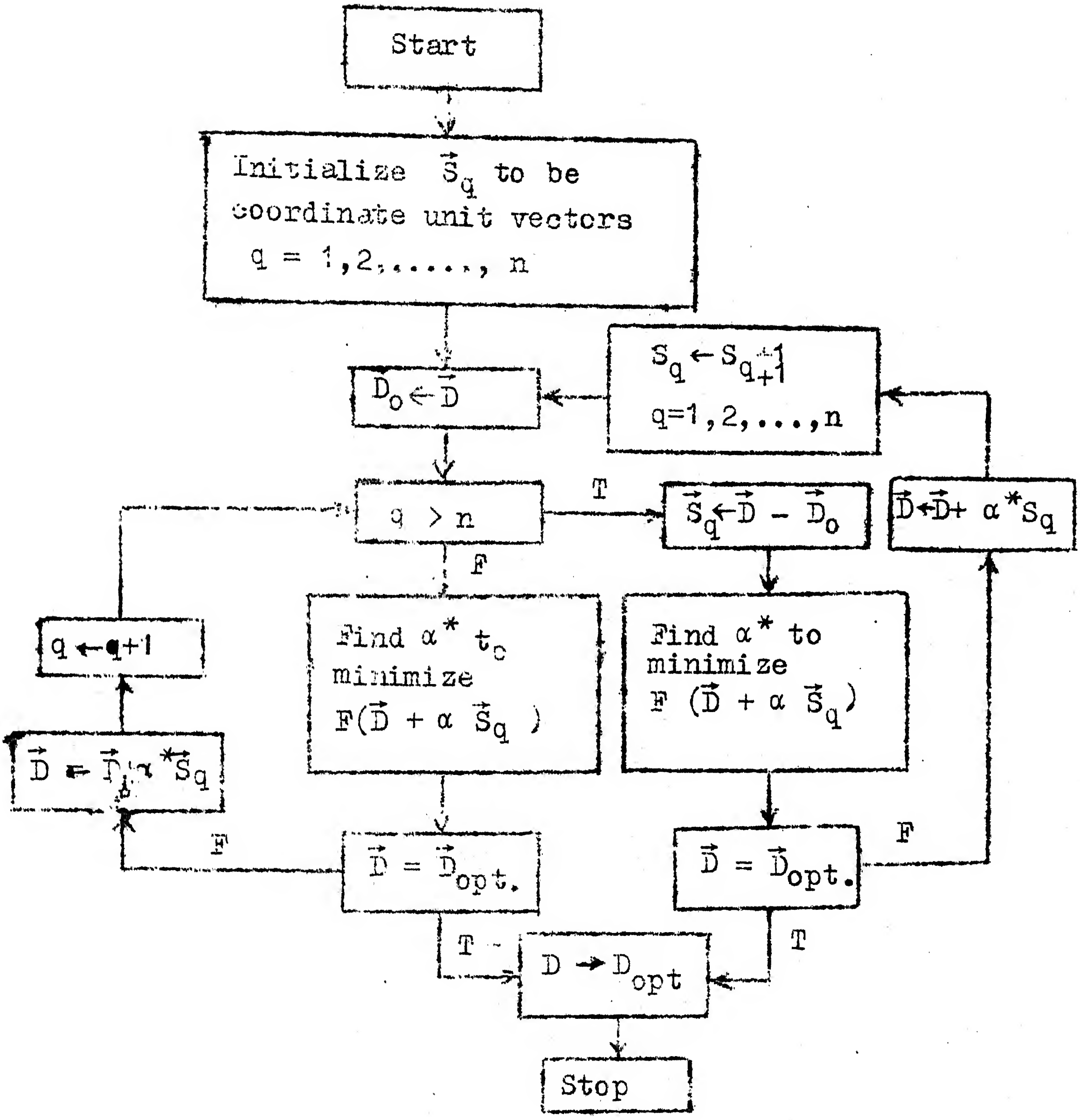
Typical two layer soil profiles



(b)

Approximate analysis for variation of E with depth

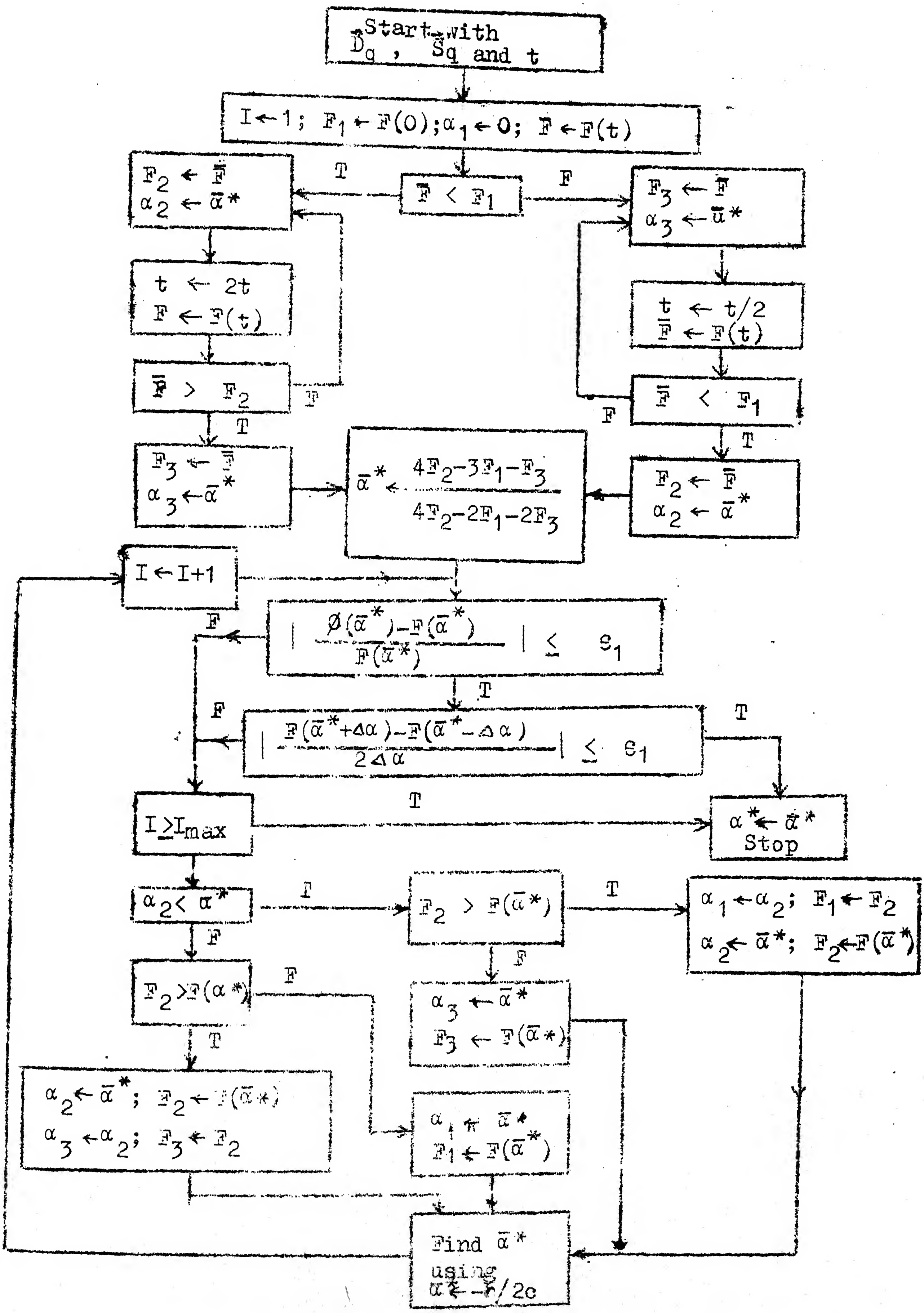
FIG. 2·3



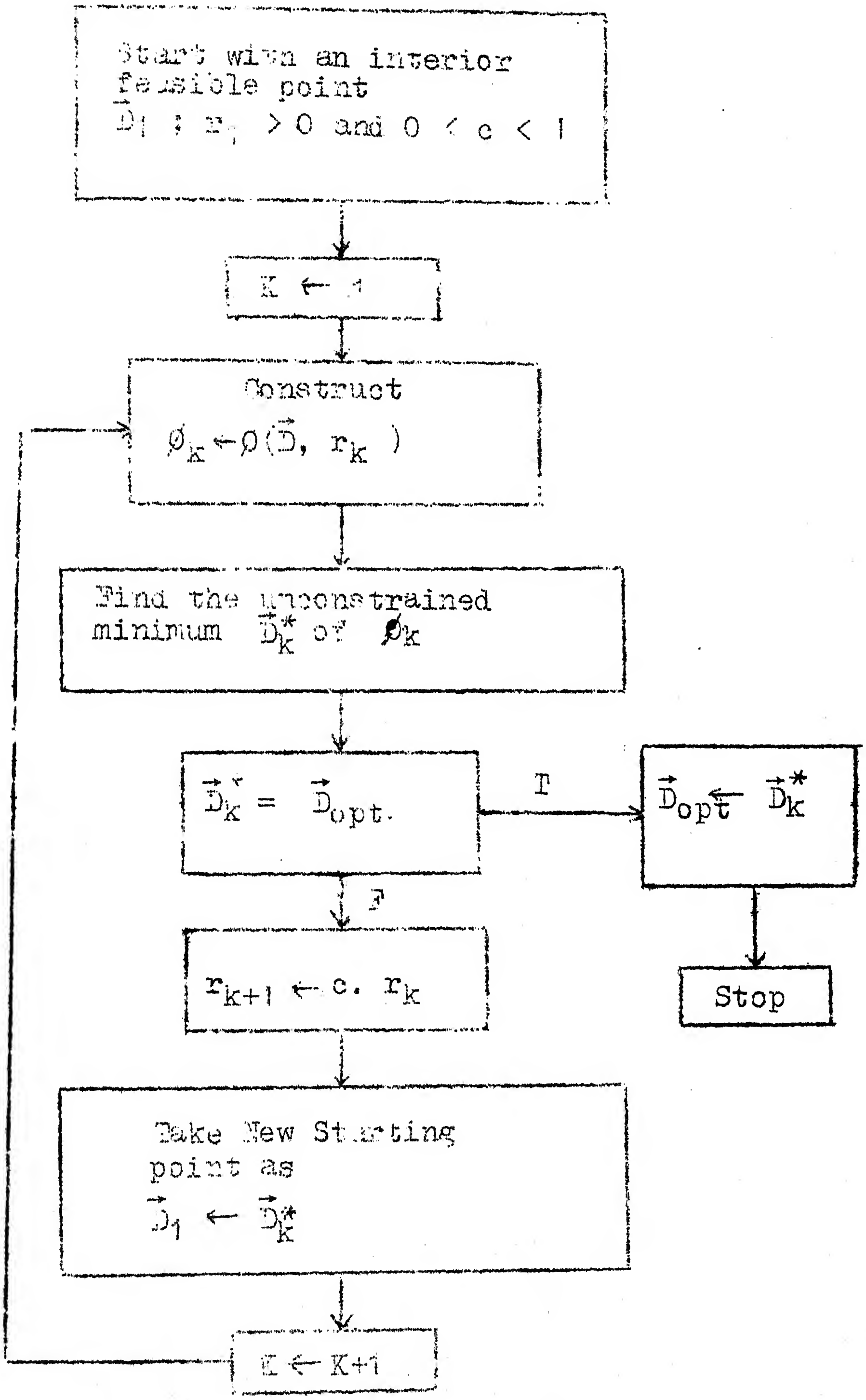
F : False

T : True

FIG. 2.4 : FLOW DIAGRAM FOR POWELL ALGORITHM



T : True
F : False



T : True

F : False

FIG. 2.6 : FLOW DIAGRAM FOR INTERIOR PENALTY FUNCTION METHOD

CHAPTER - III

SETTLEMENT CONTROLLED OPTIMAL COST DESIGN OF AN ISOLATED COLUMN FOOTING

3.1 GENERAL:

In this chapter, a general approach for the settlement controlled optimal cost design of an isolated column footing has been presented. The structural and foundation design aspects have already been discussed and presented in Chapter 2. As such, herein, only the objective function, design variables and the design constraints are identified, whenever found necessary references have been made to the appropriate sections of Chapter 2 for further details.

3.2 TOTAL QUANTITY OF THE ESTIMATED MATERIAL IN THE ISOLATED COLUMN FOOTING:

Volume of concrete in the column, VCON1 is given by

$$VCON1 = b^2 x\ (D_f - D) \tag{3.1}$$

Volume of steel in the column, VAST1 is given by the following expression

$$VAST1 = p.b^2\ (D_f - D) \tag{3.2}$$

Volume of steel used in the footing can be estimated by using the following expression,

$$VAST2 = AST2 \times L + AST3 \times B \tag{3.3}$$

Volume of concrete in the footing has two components, Fig.2.2. (a) Volume of portion A and (b) volume of portion B. Volume of the rectangular portion (A) is given by,

$$V1 = 0.15 \times L \times B$$

and volume of remaining portion (B) is given by,

$$V2 = \int_0^{D1} \left[B - \left(\frac{B-b}{D1}\right) y \right] \left[L - \left(\frac{L-d}{D1}\right) y \right] dy \qquad (3.4)$$

On integration,

$$V2 = \frac{D1}{6} (2.L.B + B.b + L.b + 2b^2) \qquad (3.5)$$

where, $D1 = D-0.15$ m and 0.15 m being the value of the thickness of the footing at the edge.

$$\text{Now, } VCON2 = V1 + V2 \qquad (3.6)$$

where , VCON2 is the volume of concrete in the footing. Thus total volume of concrete and volume of steel in the entire isolated column footing are given by

$$VCON = VCON1 + VCON2 \qquad (3.7)$$

and
$$VAST = VAST1 + VAST2 \qquad (3.8)$$

respectively.

Volume of excavation (VEX) and volume of back filling (VFIL) are estimated as follows:

$$VEX = D_f . L. B. \qquad (3.9)$$

and
$$VFIL = VEX - VCON \qquad (3.10)$$

3.3 <u>COST OF MATERIALS:</u>

The prevailing market rates are shown in Table 3.1. These rates have been used in the analysis to estimate the total cost.

3.4 STATEMENT OF THE PROBLEM:

Fig. (2.2) shows the plan and sectional elevation of an isolated column footing. Given the loads and moments on the column and the data regarding the soil profile, the problem is to determine the design variables, i.e., the dimensions of the column, dimensions of the footing and the depth of embedment in such a way that the total cost of the isolated column footing is a minimum.

Analysis is to be carried out when the soil test results are available from common type of field tests (e.g. static cone penetration test, standard penetration test) and standard laboratory test (e.g. standard triaxial test and oedometer test).

3.5 ANALYSIS:

3.5.1 Design Variables:

The following are the six design variables which control the cost of the isolated column footing:

(1) Breadth (b) of the column,

(2) Percentage of steel (p) in the column

(3) Depth of embedment (D_f)

(4) Breadth of the footing

(5) Ratio of length to breadth of the footing (m), and

(6) Thickness of the footing (D) at the centre.

3.5.3 Objective Function:

The total cost of the single isolated column footing, which includes the cost of concrete, cost of steel, cost of excavation and cost of back-filling is taken as the objective function.

The objective function 'F' is given by

$$F = C1 \times VCON + C2 \times VAST + C3 \times VEX + C4 \times VFIL \quad (3.11)$$

where, C1 = cost of concrete

C2 = cost of steel

C3 = cost of excavation

and C4 = cost of back filling.

VCON = volume of concrete in the isolated column footing

VAST = total volume of steel in the isolated column footing

VEX and VFIL are the volumes of excavations and volume of backfilling respectively.

3.6 DESIGN CONSTRAINTS:

The following restrictions are imposed on the design variables so as to ensure that the optimum design is a feasible design from the engineering point of view .

1) $EX \leq B/6$ (3.12)

2) $EY \leq L/6$ (3.13)

where, EX and EY are the eccentricities in the x and y directions respectively. This ensures that the resultant lies with in the middle third of the base.

3) $q_{max} \leq q_{all}$ (3.14)

where, q_{max} is the maximum soil pressure and is estimated by using equation (2.10) and q_{all} is the allowable bearing pressure of the soil and is estimated by using the equation (2.44) .

4) $0.0080 \leq p \leq 0.06$ (3.15)

The I.S. 456-1978 specifies that the percentage of steel(p) in the column should lie between 0.80 and 6.0 percent.

5) $\left(\frac{MUX}{MUX1}\right)^{\alpha} + \left(\frac{MUY}{MUY1}\right)^{\alpha} \leq 1.0$ (3.16)

6) $P_u \geq W$

where P_u is the ultimate load of the column given by Breslar, (Ref. Mallick and Gupta, (1979)) as

$$P_u = \frac{P_{ux} \cdot P_{uy} \cdot P_o}{P_{ux} \cdot P_o + P_{uy} \cdot P_o - P_{ux} \cdot P_{uy}} \qquad (3.17)$$

P_{ux} = ultimate load under eccentricity, EX (EY = 0)

P_{uy} = ultimate load under eccentricity, EY, (EX = 0)

P_o = Ultimate axial load capacity.

7) $D_{f1} \leq D_f \leq B$ (3.18)

where, D_{f1} = minimum depth of embedment (meters) required given by the equation, (Jain and Jai Krishna, (1973)),

$$D_{f1} = \frac{P}{\gamma}\left(\frac{1 - \sin\phi}{1 + \sin\phi}\right)^2 \qquad (3.19)$$

where, P = pressure in kg/m^2 on the soil

γ = unit weight of soil in kg/m^3

ϕ = angle of repose of the soil.

8) $\tau_v \leq \tau_a$ (3.20)

where, τ_v = transverse shear given by equation (2.24) and

τ_a = allowable shear stress in M15 concrete given by equation (2.25)

9) $q_v \leq q_c$ (3.21)

where q_v = shear stress at a distance 'd' from the face of the column given by equation (2.28) and q_c = allowable shear stress conforming to IS : 456-1978, which depends upon size of the foundation and area of longitudinal reinforcement.

10) For safe load transfer, the bearing stress cbr should satisfy the following condition,

$$\sigma_{cbr} \leq 0.45 \text{ fck} \sqrt{A1/A2} \qquad (3.22)$$

11) Total settlement 'S_e' should be less than or equal to the permissible value.

Thus for footings on clays, $S_e \leq 65$ mm (3.23)

and for footings on sands, $S_e \leq 40$ mm (3.24)

13) Differential settlement should be less than equal to the permissible value

i.e. DIFF $\leq$ 0.0035 x Span (3.25)

where DIFF is the differential settlement between any two points and Span is the distance between those two points.

14) Factor of safety (SF) against overturning moment should be greater than or equal to the specified value.

i.e. S.F. $\geq$ 1.5 (3.26)

and

15) Factor of safety against sliding should be greater than or equal to the specified value,

i.e. F.S. $\geq$ 1.5 (3.27)

The notations used in this chapter are the same as those used in the previous chapters. The total number of constraints in the present problem is 25.

3.6.1 Results and Discussions:

A software program has been developed for the settlement controlled optimal design of footings. All computations were carried out on Dec 10 computer system. To study the effectiveness and correctness of the computer program, limited results on the various aspects of the present study have been obtained and are presented as follows:

It is a well known fact that efficiency of any numerical scheme depends to a great extent on the initial starting point. As such, to study the effect of the starting point on the final solution, various starting points were taken. Such studies are also essential to know whether the obtained solution is a global optimum or not.

Table 3.2 gives the optimum value of the objective function obtained when tried with different starting points. It can be concluded from the table that the starting point does not have significant influence either on the value of the design variables or on the value of the objective function. However, it has been found that selecting the starting point closer to the optimum leads to lesser number of function evaluations. Variation in the C.P.U. time for different starting points is very small.

One of the shortcomings of the penalty method is the severe illconditioning of the unconstrained minimization problem as the value of the penalty parameter tends to its limiting value (infinity or zero). To study this aspect and to see whether minimization of $\emptyset$ leads to the minimization of F, results were obtained and presented in Fig. 3.1. The figure demonstrates that as the solution procedure progressed, when the number of function evaluations is about 1800, the minimization of $\emptyset$ leads to the minimization of F. Hence it can be concluded that there has not been any ill conditioning and the scheme works well for this class of problems.

Studies were carried out to check the convergence of the sequential unconstrained minimization technique when applied to this problem and the results are presented in Fig. (3.2). It can be observed from the Fig. (3.2) that as the penalty parameter(r) decreases from 10 to 0.1, there is a sharp decrease in the objective function value. As 'r' is further decreased, the decrease in objective function is not apprecia-ble and the solution converges to the optimum value very slowly. Stresses below the foundation have been calculated by dividing the foundation in to 100 strips. If the dimension of each strip is greater than 1/3 of the depth at which the stress is calculated, then the stress at that depth is calculated by means of linear interpolation of the stresses between the surface and the next depth. However, no interpolation is done if the depth at which the stress is required is greater than 3 times the size of each strip. The effect of the above

mentioned approximation on the final solution has been studied by dividing the foundation in to 225 strips. Table 3.3 gives the design variables and friction values for both 100 and 225 strips. From the table 3.3, it can very well be concluded that final solution is not at all significantly affected by the approximation. However, the C.P.U. time required for 225 strips is about 3 times more than that for 100 strips without yielding any significantly better solution.

Studies were also undertaken to check how much net saving one may make with the help of the computer aided optimal design as enunciated. After doing several trials, an economic and one of the best possible designs which may be obtained manually is done and the dimensions so obtained by this design is selected as the starting point for the optimization procedure. The optimum values have a significant saving over this initial design. To illustrate this, the following two problems were solved by the developed program and the results are summarised.

3.7 ILLUSTRATION:

Problem 1:

Load on the column = W = 750 kN

Moment about X-axis, MUX = 100 kN-m

Moment about Y-axis, MUY = 100 kN-m

Horizontal load about X axis, H1 = 6 kN

Horizontal load about Y axis, H2 = 6 kN

M15 concrete and 250 kN/mm^2 steel have been used.

Parameters of the soil are,

average $C = 27.0\ kN/m^2$

average $\phi = 24^\circ.6$

average corrected N value = 10

depth of water table below the footing = 1.0 m

The Table 3.4 gives the cone-penetration values at different depths.

The results and the percentage savings over the initial design are presented in Table 3.5.

Problem 2:

Problem 1 is analysed as a layered soil. Coefficient of volume change, 'mv' for each layer is computed from the relation,

$$mv = \frac{1}{f_2 \times N}$$

where, mv = coefficient of volume change in m^2/kN.

Because of the non-availability of data regarding undrained Young's modulus (E_u) for each layer, an average value of 'E_u' equal to 10000 kN/m^2, has been assumed for the present problem. Value of f_2 has been taken equal to 700. Equations (2.6c) and (2.5c) have been employed to calculate the initial and consolidation settlements respectively.

The results and percentage savings from the initial design are given in Table 3.6.

3.8 GENERAL DISCUSSIONS:

The general observations made from the observed results and its probable ~~regions~~ reasons are discussed and summarized as follows:

i) The optimum is reached when the equation (2.6) is almost **critically satisfied.**

ii) The depth of embedment 'D_f' has direct or indirect influence on the value of the objective function in the following manner:

a) Increase in the depth of embedment increases the allowing bearing pressure significantly. Higher the value of the depth of embedment (D_f), the lower is the plan area of the footing required. This in turn leads to a considerable saving in the area of concrete required in the footing.

b) Total settlement also decreases with the increase in the depth of embedment (D_f). The settlement, and not the bearing capacity is a major factor which controls the required plan dimensions of the footing. Besides, the decrease in the total settlement also decreases the differential settlement between the different points in the footing. Thus the decrease in settlement contributes to the saving in the plan area of concrete in the footing.

c) Footings are never placed on the ground surface. They are always placed at a depth (D_f) below the ground surface. Fox (1948), has given the coefficient (F3) which should be multiplied with the computed surface settlement so as to obtain

the actual settlement at that depth. This coefficient is also found to decrease the settlement with the increase in (D_f/B) ratio. This in turn reduces the actual settlement which has an effect over the plan area of the footing as discussed in (b).

All the factors (a), (b) and (c) contribute towards the saving in the plan area of the footing.

However, the increase in the depth of embedment (D_f) increases the volume of concrete and volume of steel in the column. Besides, the moment coming on the column due to a horizontal load on the column also increases with the increase in the depth of embedment (D_f). But the saving in the volume of concrete in the footing is relatively much higher than the extra quantity of concrete used in the column. It should also be noted that the depth of embedment (D_f) increases the allowable bearing pressure to a maximum value equal to, twice the allowable bearing pressure at the surface. The extra cost involved due to the increase in the depth of embedment (D_f) on the cost of excevation and cost of filling are relatively low when compared with the cost of concrete saved in the footing. Consequently, the present work enables to select the optimum value of depth of foundation (D_f) required for a particular soil, and for a particular foundation load so as to save the area of concrete in the footing as much as possible.

iii) It has been observed that differential settlement between any two points in a footing actually controls the minimum dimensions of the footing required so as to keep the

differential settlement with in the allowable limits. This argument is better justified especially when a foundation is subjected to eccentric loads.

iv) The overall depth of the foundation is adjusted along with the plan dimensions of the footing in such a way that volume of concrete used in the footing is a minimum. Besides, area of steel in the footing will also be taken in to consideration while adjusting the value of D.

3.9 CONCLUSIONS:

(1) A software ~~program~~ for the optimal limit state design of an isolated column footing subjected to bi-axial moment has been developed.

(2) The program developed is a general one and can handle both homogeneous and non-homogeneous soil profiles.

(3) Input for the soil-data can be the test results of standard penetration test, cone penetration test and triaxial tests.

(4) The user is only required to feed the soil data, forces on the column and the characteristic strengths of steel and concrete used.

(5) The developed program ensures atleast 8-10 percent saving in the total cost of the structure.

The program will be available for reference.

TABLE 3.1 : PREVALLING COST OF MATERIALS

Item	Cost per/m^3 in rupees
Concrete	600.00
Steel	52700.00
Excavation	~~3.50~~ 7.00
Filling	~~7.00~~ 3.50

TABLE 3.2 : EFFECT OF STARTING POINT ON THE OBJECTIVE FUNCTION

Sl. No.		Value of design variables						Total number of function evaluations	Value of objective function in rupees
		b	p	D_f	B	m=L/B	D		
1	Initial	0.550	0.096	1.20	6.00	1.206	0.500	1751	12870.00
	Final	0.456	0.014	2.03	3.46	1.005	0.633		4576.41
2	Initial	0.500	0.010	1.60	3.65	0.955	0.580	1251	5076.97
	Final	0.450	0.0142	2.01	3.45	1.008	0.635		4583.82
3	Initial	0.700	0.015	1.80	3.80	0.920	0.700	1570	5976.25
	Final	0.453	0.0139	2.07	3.45	1.030	0.623		4584.75

TABLE 3.3 : EFFECT OF SIZE OF STRIP ON THE OPTIMUM SOLUTION

Sl. No.	Design variables						Number of strips
1	0.450	0.0142	2.01	3.45	1.008	0.635	100
2	0.452	0.0137	2.00	3.44	1.010	0.631	225

TABLE 3.4 : TEST RESULTS OF CONE PENETRATION TEST

Depth in metres	Cone penetration value in kN/m^2	Thickness of each lyer in metres
0.5	2500	1.0
1.5	2800	1.0
2.5	3200	1.0
3.5	3600	1.0
4.5	3900	1.0
6.0	4200	1.0

TABLE 3.5 : OPTIMUM DESIGN VARIABLES AND PERCENTAGE SAVING IN COST- HOMOGENEOUS SOIL

Sl. No.	Value of the Variables						Function value (cost) in Rs.	o/o Savings in Cost
	b	p	D_f	B	m-L/B	D		
1. Initial	0.50	0.0100	1.60	3.65	0.955	0.580	5076.97	10.93
2. Optimum	0.45	0.0142	2.01	3.45	1.008	0.633	4576.41	

TABLE 3.6 : OPTIMUM DESIGN VARIABLES AND PERCENTAGE SAVING IN TWO-LAYERED SOIL

Sl. No.		Value of variables						Function Value in Rupees	Percentage saving in cost
		b	p	D_2	β	m	D		
1.	Initial	0.50	0.0100	1.60	3.650	0.955	0.58	5076.97	9.7
2.	Optimum	0.46	0.0147	1.35	3.505	0.985	0.61	4627.30	

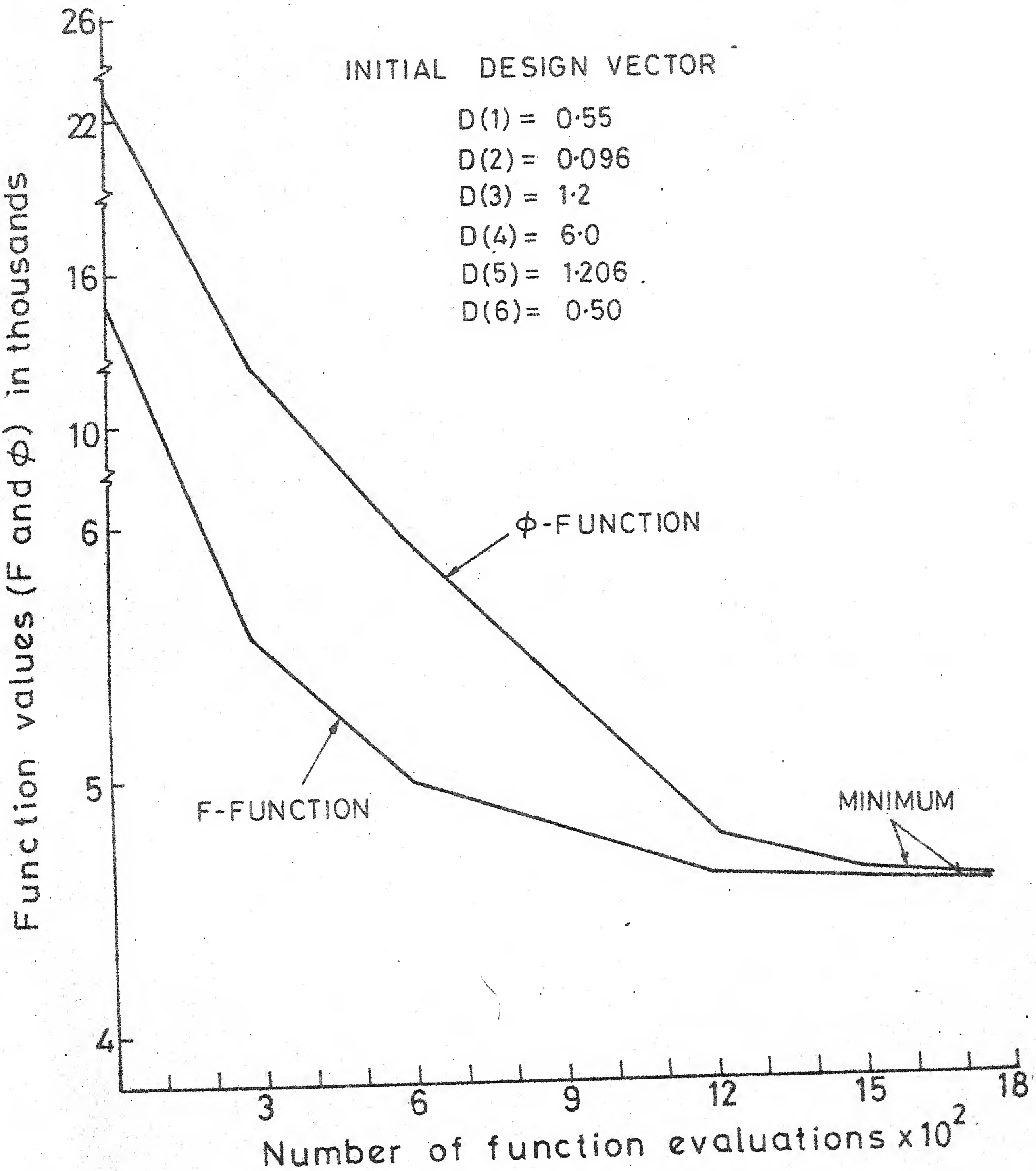


FIG.3·1 PATH OF F AND φ FUNCTIONS FROM STARTING POINTS

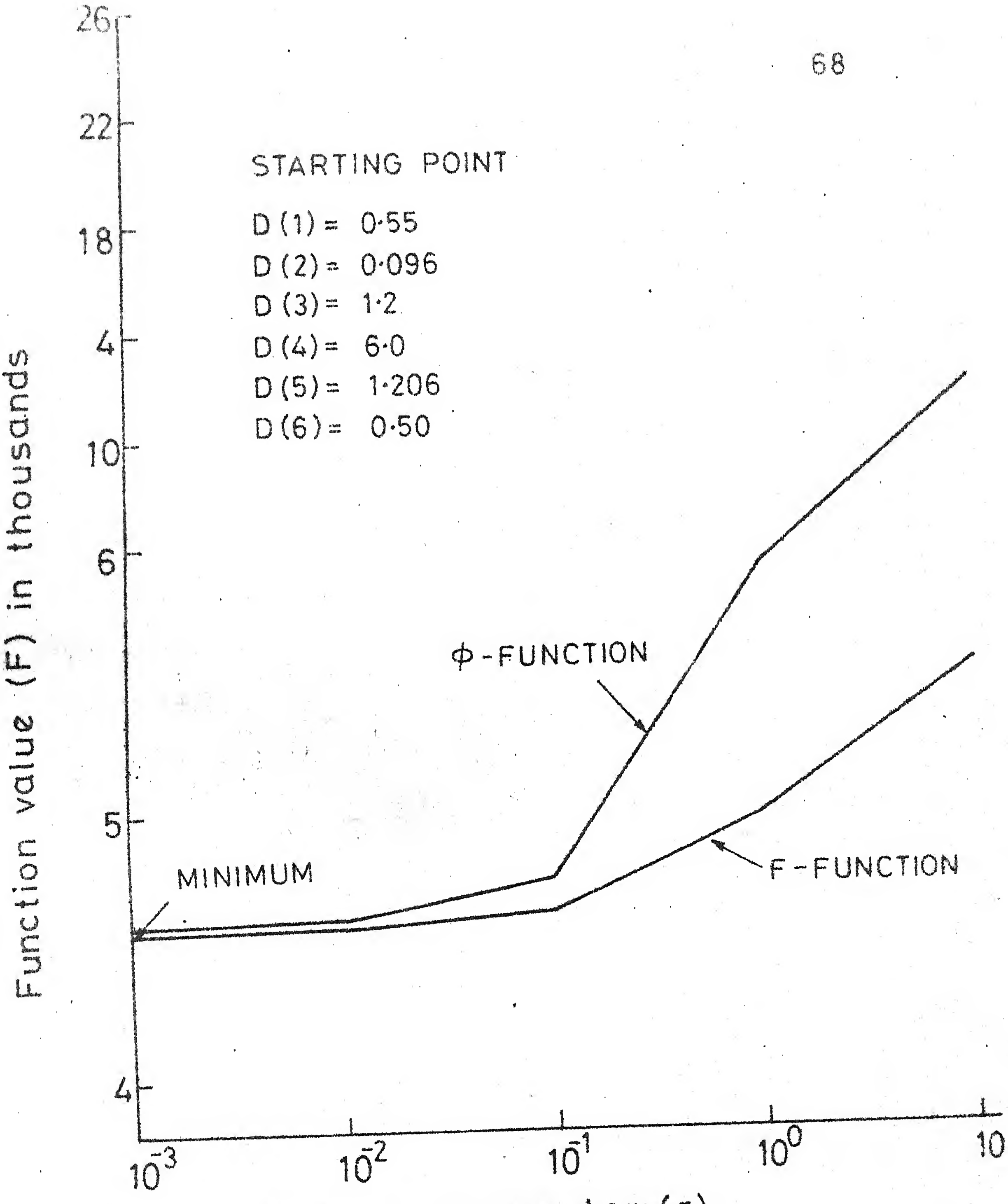


FIG. 3.2 VARIATION OF F WITH PENALTY PARAMETER

CHAPTER - IV

OPTIMUM PLAN DIMENSIONING OF A GROUP OF FOOTINGS

4.1 GENERAL PRINCIPLES OF DESIGN:

The average ultimate bearing capacity of the footings in a group is greater than the ultimate bearing capacity of an isolated footing of the same size. When the footings are placed in a group, the individual foundation failure mechanism becomes distorted due to the overlapping of the failure surfaces.

The settlement for a given load intensity decreases as the centre to centre spacing between footings decreases, Singh et al. (1973) . As the footings sink in to the ground, the soil compresses to a certain distance on both sides of the footing. This results in an increase in the strength of soil. The increase of strength of soil plays its role in the settlement characteristics, Singh, et.al.(1973).

The ultimate bearing capacity and settlement of adjacent square footings on sand are not significantly affected when the spacing between them is more than five times the width of the footing, Singh et.al (1973).

Based on the experimental results, the following factors have been defined in order to account for the change in behaviour of a footing when placed in a group (Singh et al.,1973).

(a) Interference efficiency factor F_B for bearing capacity:

It is the ratio of the ultimate bearing capacity of the footing group to that of an equal number of identical isolated footings (Stuart, 1962), i.e,

$$F_B = \frac{q_f(\text{group})}{n \times q_f(\text{isolated})} \qquad (4.1)$$

where, q_f = ultimate bearing capacity of the soil,

n = total number of footings in the group.

The ultimate bearing capacity of a smooth rectangular footing can be predicted by introducing the interference factor (FB) for bearing capacity in the Terzaghis ultimate bearing capacity relationship as follows (Singh et al.1973).

$$F_B = 2.25 - 0.31 \left(\frac{S}{B}\right) \text{ for } S/B \leq 3.25 \qquad (4.2a)$$

$$F_B = 1.69 - 0.135\left(\frac{S}{B}\right) \text{ for } S/B \geq 3.25 \leq 5.0 \qquad (4.2b)$$

and $$F_B = 1.04, \text{ for } S/B = 5 \qquad (4.2c)$$

(b) Interference efficiency factor (F_S) for settlement:

It is the ratio of the settlement of the footing group at a given intensity of pressure to that of an identical isolated footing at the same intensity of pressure multiplied by the number of footings in the group.

$$F_S = \frac{\text{Settlement (group)}}{n \times \text{settlement(isolated)}} \qquad (4.3)$$

It has been observed that the factor F_S increases almost linearly with the increase in S/B ratio, Singh et al.(1973) 'F_S' is given by

$$F_S = 0.4 + 0.1 \times S/B , \text{ for } S/B \leq 5 \qquad (4.4)$$

In the above expressions,

B = width of footings, and

S = centre to centre distance between the adjacent footings.

All the investigations on the interference between the adjacent footings have been done only when the footings lie along a line. However, no study has been made on the behaviour of a footing when it is surrounded by a group of other footings. Therefore, in the present work, it is assumed that a footing is affected by only one of the adjacent footings whichever gives the maximum S/B ratio.

It should be noted that in the case of clayey soils, it is rather more reasonable to assume that the properties of the soils are not much affected due to the presence of the adjacent footings.

It is well known that the settlement generally governs the design of foundations on sands. Hence ignoring the effect of interference on the bearing capacity does not make the design conservative.

The present work comprises of finding out the optimum plan dimensions of a group of axially loaded footings for a given depth of foundation.

Debeer's method corrected by Meyerhof (1965) has been employed to find out the settlement below the footing.

The stresses and displacements caused due to load on one footing below the other footing plays a significant role on the shape and size of the other footings. The serious limitation of the Schmertmann's method of determining the settlement lies in the fact that Schmertmann's method can determine the settlement due to the footing load below itself only. Hence Debeer's method is used for calculating the settlement.

4.2 OBJECTIVE FUNCTION, DESIGN VARIABLES AND CONSTRAINTS:

It is well known that cost of a piece of land is rising at a very fast rate especially in urban areas. The idea of an engineer should be to optimise the cost of a structure in all possible ways . Saving in the land area can be achieved to some extent by optimizing the plan area required for a group of footings. This will in addition, ensure an equivalent saving in the plan area of concrete.

A group of foundations are generally designed for an equal settlement or an equal allowable bearing pressure . Considerable saving can be made by designing a foundation group for an allowable differential settlement instead of designing the foundations for an equal settlement. It is manually difficult to design a group of foundations such that differential settlement between any two foundations in a group is nearly equal to the maximum allowable differential settlement This can be done easily in an useful way by using the mathematical programming techniques. This is the basic idea behind taking this problem as an optimization problem.

4.2.1 Statement of the Problem:

Given the loads on each foundation, the soil properties and the allowable total and differential settlement, the problem is to determine the optimum size of each foundation in a group.

Objective function:

The total plan area of all the foundations has been taken as the objective function. The objective function is given by

$$F = \sum_{i=1}^{n} L_i \; B_i \tag{4.5}$$

where,

L_i = Length of ith foundation

Bi = Breadth of ith foundation

n = Number of foundationsin the group.

4.2.2 Constraints:

Some restrictions are required to be placed so as to ensure that the foundation area provided is adequate from the safety point of view .

1) $q_{max_i} \leq q_{all_i}$ (4.6)

where, q_{max_i} = maximum pressure on the soil given by

$$q_{max_i} = \frac{W_i}{L_i \cdot B_i} \tag{4.7}$$

where, W_i = is the load on the ith footing

q_{all_i} = allowable bearing pressure on the ith foundation given by equation (2.44).

TABLE 4.1 : EFFECT OF STARTING POINT ON THE OBJECTIVE FUNCTION

Sl. No.		Design variables in metres											
		L_1	B_1	L_2	B_2	L_3	B_3	L_4	B_4	L_5	B_5	L_6	B_6
1.	Initial	3.900	3.900	3.900	3.900	3.900	3.900	3.900	3.900	3.900	3.900	3.9	3.9
	Final	1.520	1.530	1.980	1.790	1.330	1.550	1.750	2.003	1.98	2.110	1.76	1.94
2.	Initial	2.014	2.014	2.470	2.470	2.014	2.014	2.470	2.470	2.700	2.470	2.47	2.47
	Final	1.518	1.505	1.973	1.823	1.407	1.586	1.705	2.090	1.912	2.446	1.71	2.[illegible]8

		L_7	B_7	L_8	B_8	L_9	B_9	Function value in m^2	Number of function evaluations
1.	Initial	3.900	3.900	3.900	3.900	3.900	3.900	136.39	3711
	Final	1.520	1.530	1.980	1.970	1.380	1.550	27.97	
2.	Initial	2.014	2.014	2.470	2.470	2.019	2.014	47.91	3385
	Final	1.518	1.505	1.993	1.823	1.407	1.586	28.12	

To check whether there is any illconditioning of the unconstrained minimization problem with the decreasing value of the penalty parameter, results were obtained and are presented in Fig. 4.1. The figure shows that minimization of the ϕ function leads to the minimization of F function and as such it can be concluded that no ill conditioning has occurred leading to any premature termination of the algorithm. When the number of function evaluations is about 3800, the minimization of ϕ leads to the minimization of F function, Fig. 4.2.

Optimum plan dimensioning of a group of footings is generally done either on the basis of equal bearing pressure or on equal settlement. This approach is too conservative. Considerable saving can be achieved if all the footings are designed in such a way that the differential settlement between any two footings is almost equal to the maximum allowable differential settlement. This design becomes tedious and cumbersome when the number of footings in the group is more. However, this kind of a problem can be easily handled with the aid of a computer, using mathematical prgramming techniques.

To illustrate the amount of saving that one can achieve by the developed methodology in present study over conventional approach, the following problems have been solved.

4.3.1 Illustration:

Referring to Fig. (4.3), the loads on the footings

W1 = 800 ; W3 = 800 ; W5 = 1500 ; W7 = 800 and W9= 800

W2 = 1200; W4 = 1200; W6 = 1200; W8 = 1200.

Properties of the soil:

Average $C = 0.0$

Average $\phi = 32^{o}$

Average corrected $N = 22$

Equal spacing between the footings 7.5 m.

The cone penetration values are tabulated in the Table 4.2. Following the procedure as suggested by Teng (1968), an initial design is obtained such that all the footings are subjected to equal bearing pressure. This ensures that there is no differential settlement between any two footings. The dimensions obtained from this initial design are taken as the starting point for the optimization problem. The saving obtained by the optimum design over that of the initial is presented in Table 4.3. It can be seen that there is a substantial net saving in the plan area (41.3 percent).

In case of clayey soils, the most reasonable assumption we can make is that the soil properties are not changed due to the interference between the adjacent footings. It is known that the properties of clayey soils are not changed like that of sand on densification. To enunciate the savings that one can make while designing a group of shallow footings on clay, the following problem has been solved by the developed program.

Problem 2:

Referring to Fig.(4.3), the loads on each footing in kN are as follows:

Loads in kN

W1 = 750 ; W3 = 750 ; W5 = 1000 ; W7 = 750 and W9= 750

W2 = 800 ; W4 = 800 ; W6 = 800 ; W8 = 800

Properties of the soil are as follows:

Average $C = 27\ kN/m^2$

$\phi = 0.0$

Equal spacing between the footings 7.5 m.

Cone penetration values are the same as those given in Table 3.5 . The average corrected. N = 10. The depth of foundation = 1.8 m.

The results and the net savings in the plan area over the initial mannual design are shown in Table 4.4 . In this case also the net saving in the total plan area is substantial (37.3 percent).

4.3.2 <u>Influence of Depth of Foundation on the Total Plan Area</u>

It is a well known fact that depth of embedment (D_f) increases the allowable bearing pressure and decreases the tot settlement. Depth of embedment (D_f) also has a signfficant influence on the required plan dimensions of footings. To illustrate this, optimum total plan area of a group of footing are determined for different depth of embedments and the variation of optimum total plan area with D_f is shown in Fig. (4.4).

It can be observed that as the depth of foundation (D_f increased from 1.2 m, to 2.0 m the net decrease in the total plan area (F) is quite substantial (6.5 m^2); however, with th

increase in $D_f > 1.5$ m , The rate of decrease in F decreases. Beyond 2 m the effect of depth of foundation on the total plan area is not appreciable. The depth of foundation could have been incorporated in the formulation as a design variable but was not so considered as from various considerations , the normal practice is to get the plan area for a given depth of foundation. However, as the study indicates, this should also be considered as a design variable for a realistic analysis to arrive at the global optimum value as the solution that one gets for a given depth of foundation can at best is a local optimum. Settlement below each footing is presented in Table 4.5.

4.4 CONCLUSIONS:

1. A general optimization scheme has been outlined for the optimal plan dimensioning of a group of footings and it has been demonstrated that the developed algorithm is quite efficient is arriving at an optimal solution.
2. Very significant saving in the total plan area of a group of footings can be achieved by following the scheme.
3. The obtained solution is independent of the starting point.

TABLE 4.1: EFFECT OF STARTING POINT ON THE OBJECTIVE FUNCTION

Sl. No.		Design variables in metres											
		L_1	B_1	L_2	B_2	L_3	B_3	L_4	B_4	L_5	B_5	L_6	B_6
1.	Initial	3.900	3.900	3.900	3.900	3.900	3.900	3.900	3.900	3.900	3.900	3.9	3.
	Final	1.520	1.530	1.980	1.790	1.330	1.550	1.750	2.003	1.98	2.110	1.76	1.
2.	Initial	2.014	2.014	2.470	2.470	2.014	2.014	2.470	2.470	2.700	2.470	2.47	2.
	Final	1.518	1.505	1.973	1.823	1.407	1.586	1.705	2.090	1.912	2.416	1.71	2.

Sl. No.		L_7	B_7	L_8	B_8	L_9	B_9	Function value in m^2	Number of function evaluations
1.	Initial	3.900	3.900	3.900	3.900	3.900	3.900	136.39	3711
	Final	**1.520**	**1.530**	**1.980**	**1.970**	**1.380**	1.550	27.97	
2.	Initial	2.014	2.014	2.470	2.470	2.019	2.014	47.91	3385
	Final	1.518	1.505	1.993	1.823	1.407	1.586	28.12	

TABLE 4.2 : CONE PENETRATION TEST IN SAND

Layer	Thickness of layer in metres	Cone value in kN/m^2
A	2.5	3000
B	2.5	8000
C	2.5	10000
D	2.5	10000
E	2.5	12000
F	2.5	12000

TABLE 4.3 : PERCENTAGE SAVING IN THE TOTAL PLAN AREA OF CONCRETE OF A GROUP OF FOOTINGS ON SAND

Sl. No.		Design variables in metres											
		L_1	B_1	L_2	B_2	L_3	B_3	L_4	B_4	L_5	B_5	L_6	B_6
1.	Initial	2.014	2.014	2.470	2.470	2.014	2.014	2.470	2.47	2.700	2.700	2.47	2.014
	Final	1.518	1.505	1.993	1.823	1.407	1.586	1.705	2.09	1.912	2.416	1.71	2.080

Sl. No.		L_7	B_7	L_8	B_8	L_9	B_9	Function value in m^2	Percentage saving in area
1.	Initial	2.014	2.014	2.470	2.470	2.019	2.014	47.91	41.3
	Final	1.518	1.505	1.973	1.823	1.407	1.586	23.12	

TABLE 4.4 : PERCENTAGE SAVING IN THE TOTAL PLAN AREA OF CONCRETE OF A GROUP OF FOOTINGS ON CLAY

Sl. No.		Design variables in m^2									
		L_1	B_1	L_2	B_2	L_3	B_3	L_4	B_4	L_5	B_5
1.	Initial	1.86	1.86	1.920	1.920	1.860	1.860	1.52	1.52	2.15	2.15
	Final	1.35	1.46	1.417	1.625	1.364	1.406	1.419	1.62	1.716	1.78

Sl. No.		L_6	B_6	L_7	B_7	L_8	B_8	L_9	B_9	Function value in m^2	Percentage saving in area
1.	Initial	1.52	1.52	1.86	1.86	1.52	1.92	1.86	1.86	33.21	37.28
	Final	1.41	1.68	1.35	1.46	1.417	1.625	1.364	1.455	20.28	

TABLE 4.5 : SETTLEMENT BELOW EACH FOOTING ON SAND

Number of the footing	Settlement below the initial design (mm)	Settlement below the final design (mm)
1	30.870	37.23
2	31.309	35.36
3	31.020	37.10
4	31.324	35.60
5	31.520	34.40
6	31.328	35.63
7	31.078	37.208
8	31.297	35.330
9	30.790	37.287

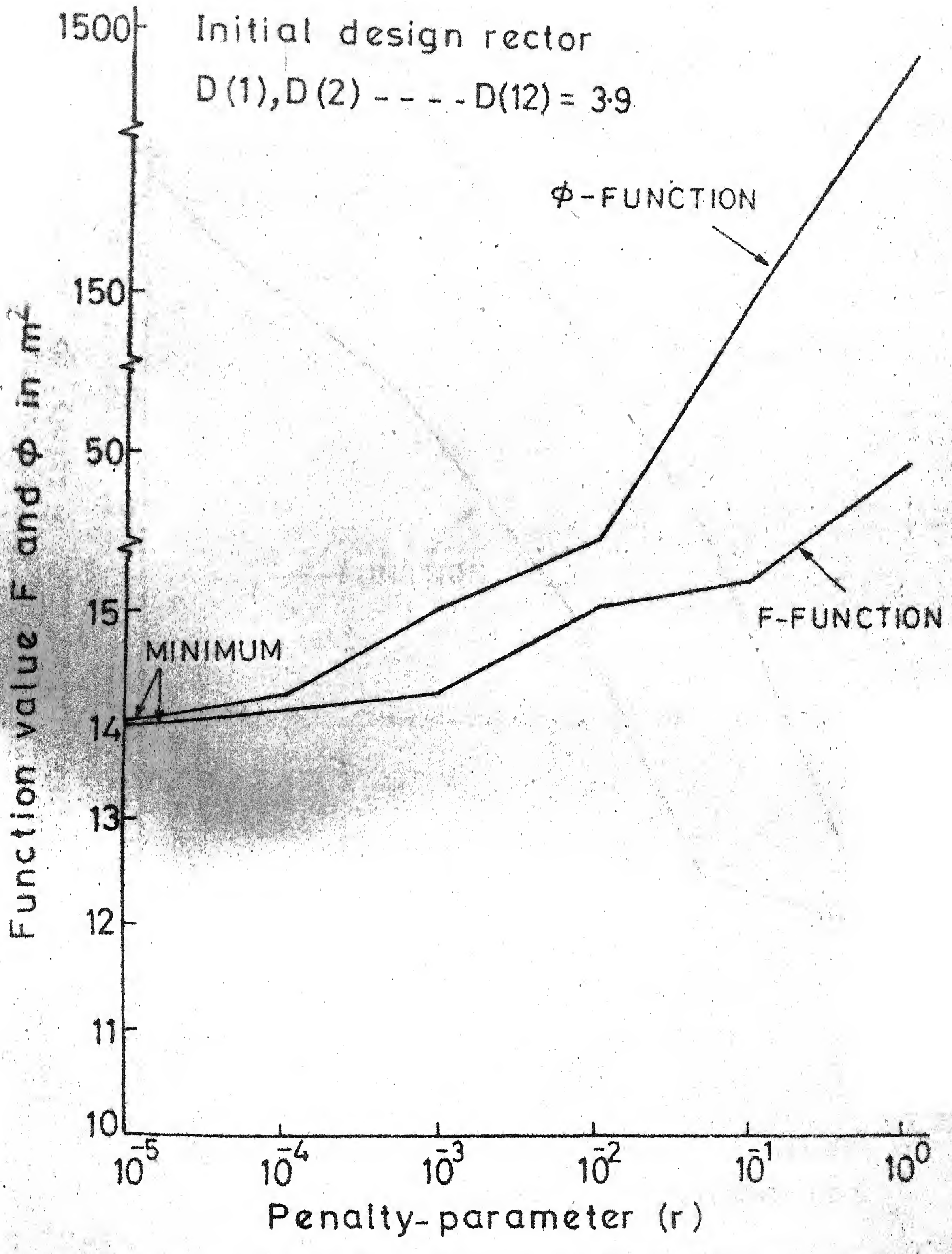


FIG. 4·1 DIAGRAM SHOWING VARIATION OF F AND Φ WITH r

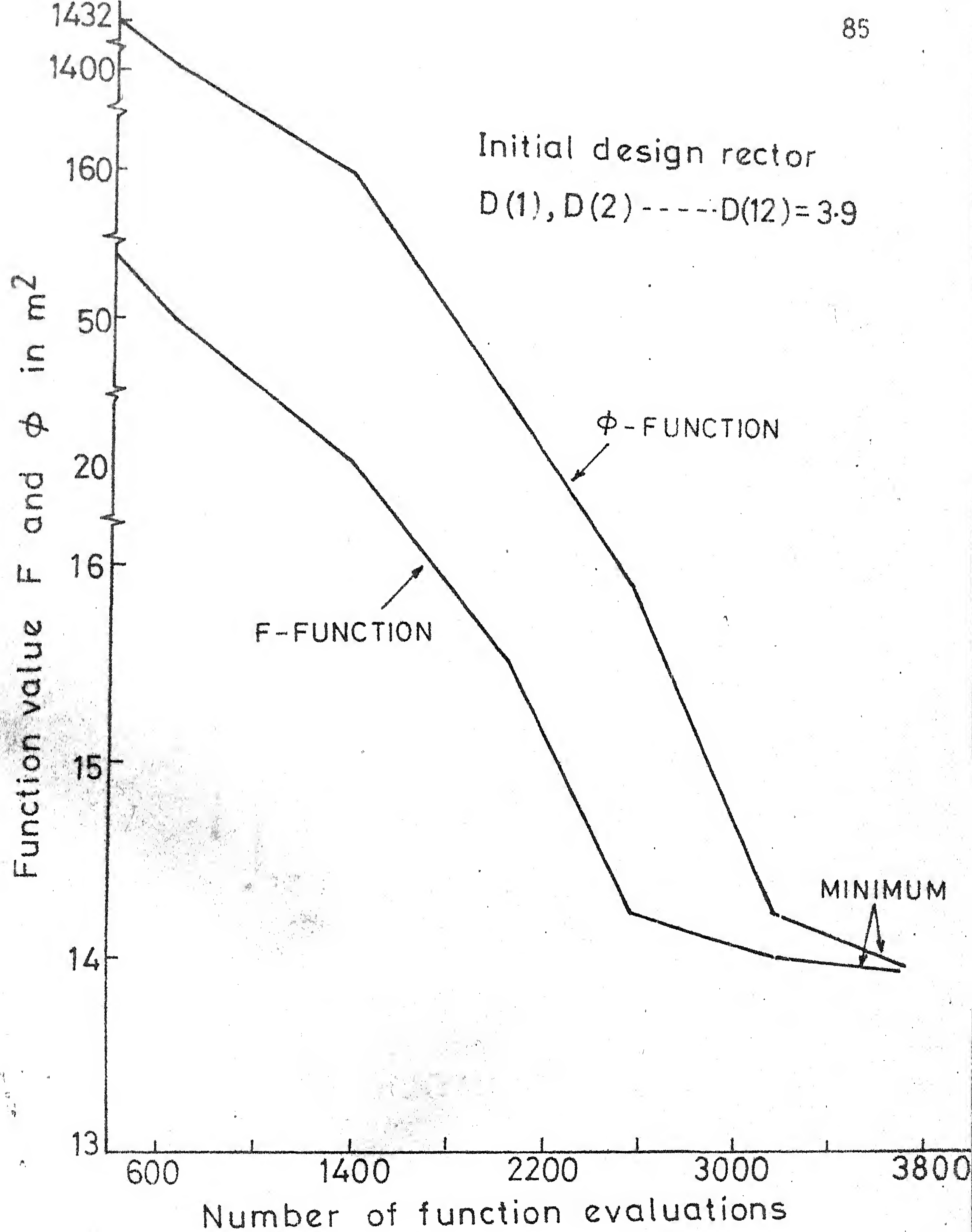


FIG.4·2 PATH OF F AND ϕ FUNCTIONS FROM STARTING POINTS

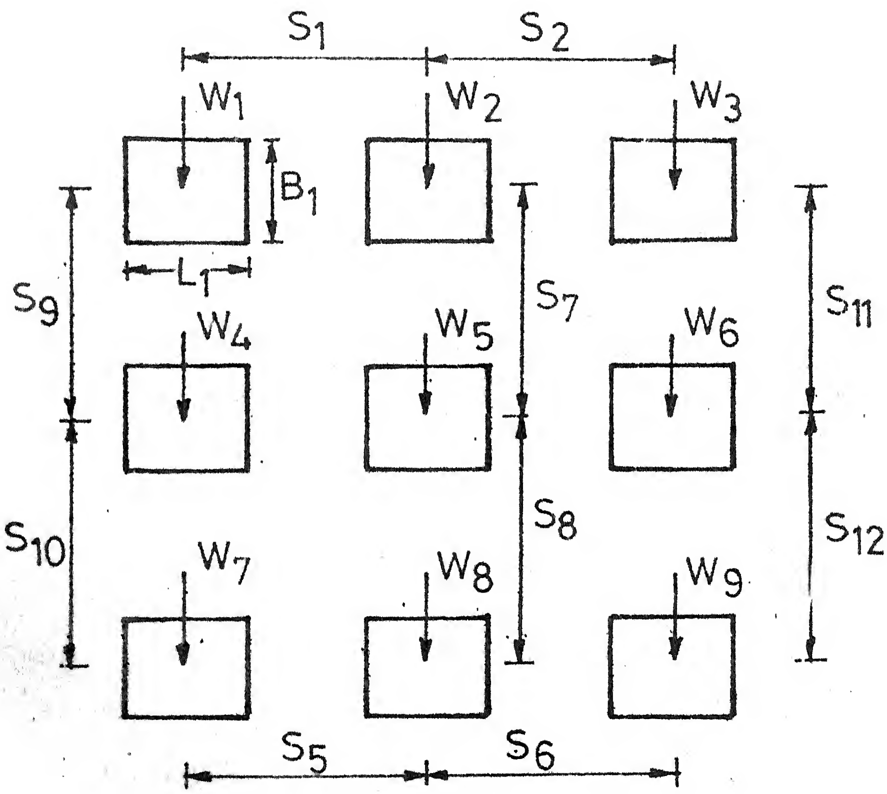


FIG. 4·3 PLAN SHOWING THE GROUP OF FOOTINGS

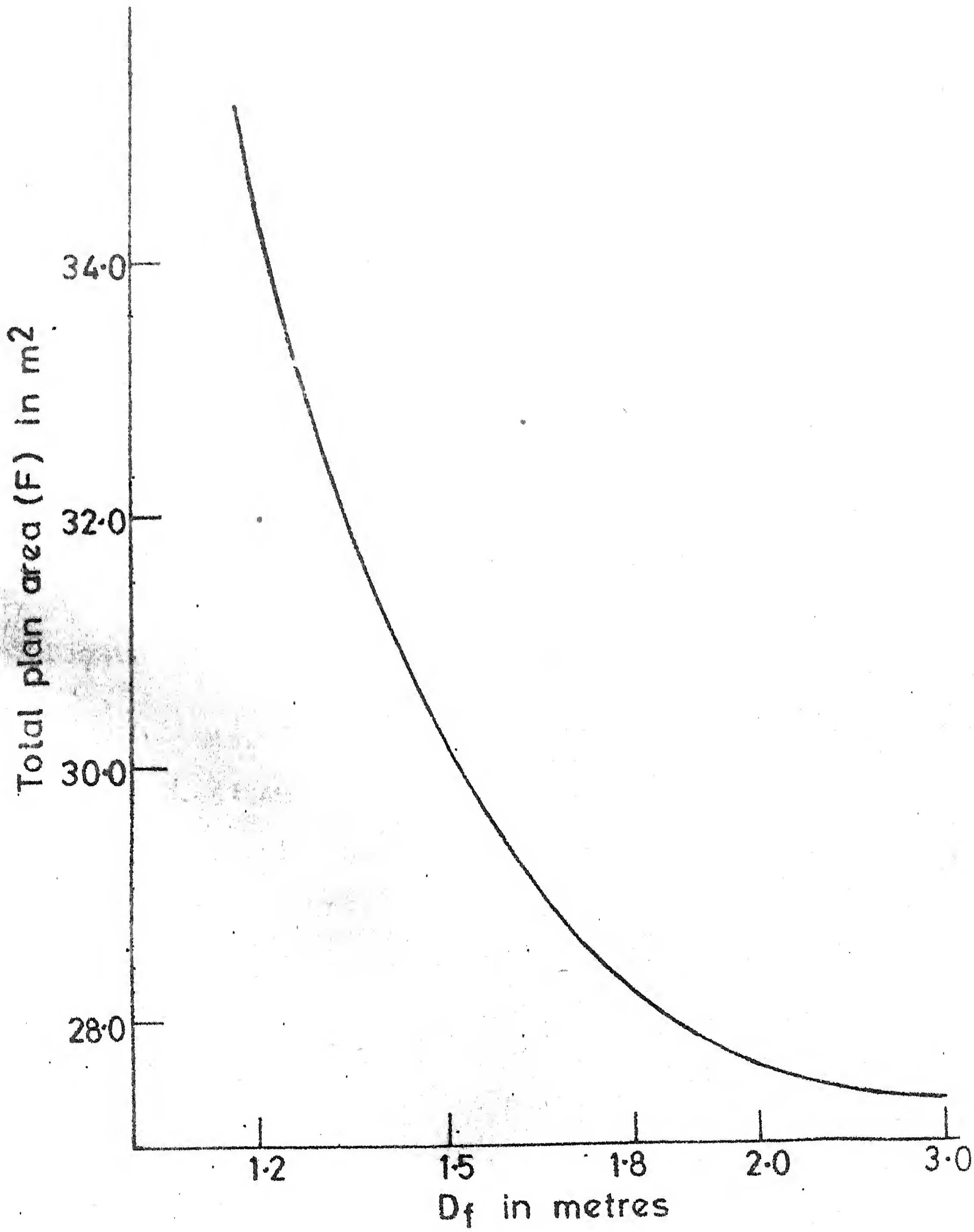


FIG. 4.4 EFFECT OF D_f ON THE TOTAL PLAN AREA OF A GROUP OF FOOTINGS

REFERENCES

Alam Singh, B.C. Punmia and M.L. Ohri, 'Interference Between Adjacent Square Footings on Cohesionless Soil

S.S. Bhavikatti and Hegde, 'Optimum Design of Isolated Column Footing Using Sequential Linear Programming', International Conference on Computer Applications in Civil Engineering (U.Of Roorkee, Oct, 1979).

Brown, J.D. and Meyerhof, (1969), 'Exptal Study of Bearing Capacity in Layered Clays,' Proceedings of 7th International Conference of Foundations and Engineering, Mexico-city, Vol.2, pp. 45-51.

Button, S.J. (1953), 'The Bearing Capacity of Footings on a Two Layered Cohesive Subsoil,' Proceedings of 3rd International Conference of Soil Mechanics and Foundation Engg., Zurich, Vol.1, pp. 332-335.

Debeer, E.E. (1965), 'Bearing Capacity and Settlement of Shallow Footings on Sand', Proc. Symposium on Bearing Capacity and Settlement of Foundations, Duke University, pp. 15-33.

Desai, C.S. and Reese, L.C. (1970), 'Analysis of Circular Footings on Layered Soils,' Proceedings of A.S.C.E., Journal of Soil Mechanics and Foundation Engineering, Div. 90, SM4, pp. 1289-1310.

Fox, E.N. (1948), 'The Mean Elastic Settlement of a Uniformly Loaded Area at a Depth Below the Ground Surface, 2nd ICSMFE, Vol.1, pp. 129-132.

Gibbson, R.E. (1967), 'Some Results Concerning Displacements and Stresses in a Nonhomogeneous Elastic Half Space,' Geotechnique 17, 1, p. 58-67.

Giroud, J.P. (1970a), 'Stresses Under a Linearly Loaded Rectangular Area', J.SM Div. ASCE 96, 1, p.263-268.

Hansen, J.B. (1970), 'A Revised and Extended Formula for Bearing Capacity', Danish Geotechnical Institute, Bull. 28, Copenhagen, 21pp.

Joseph, E. Bowles, 'Foundation Analysis and Design'.

Mac Donald, D.H., and A.W. Skempton (1955), 'A Survey of Comparisons Between Calculated and Observed Settlements of Structures on Clay', Conference on Correlation of Calculated and Observed Stresses and Displacements, ICE, London, pp. 318-337.

Meyerhof, G.G. (1953), 'Bearing Capacity of Foundations Under Eccentric and Inclined Loads, Proceedings of 3rd International Conference of Soil Mechanics and Foundation Engg., Zurich, Vol. V, pp. 440-445.

Meyerhof, G.G. (1956), 'Penetration Tests and Bearing Capacity of Cohesionless Soils, JSMFD, ASCE, Vol.82, SM 1, pp.1-19.

Myslivec, A., and Kyser, Z. (1973), 'Interaction of Neighbouring Foundations', Proceedings of International Conference on Soil Mechanics and Foundation Engg., Moscow, Vol.1, 3, p. 181,

N.E. Simons, B.K. Menzies, 'A Short Course in Foundation Engineering', E.L.B.S. (1977).

S.S.Rao, 'Optimization Theory and Applications', Wiley Eastern Company Limited.

S.K. Mallick and A.P. Gupta, 'Reinforced Concrete ', Oxford and I.B.H. Publishing Co. New Delhi.

R.L. Fox, 'Optimization Methods for Engineering Design', Addison Wesley, Reading Mass, 1971.

Schmertmann (1970), 'Static Cone to Compute Static Settlement on Sand', A.S.C.E., 96, No. SM3.

Sivareddy, A. and Srinivasan, R.J. (1967), 'Bearing Capacity of Footings on Layered Clays', Proc. ASCE, Journal of S.M. and F.E. Division 93, No. SM2, pp. 83-89.

Stein Brenner, W. (1934), Tafeln Zur Setzurgsberechnung, Die Strasse Vol.1, Oct., pp. 121-124.

Stuart, J.G. (1962), 'Interference Between Footing with Special Reference to Surface Footings on Sand, Geotechnique, Vol.12, No.1, pp. 15-23.

K.S. Subbarao, N. Norayana Rao and B.V. Ranganatham, Indian Geotechnical Journal, Vol.2, Jan. 1972.

Vesic, A.S. (1970), 'Bearing Capacity of a Two Layered Soil', JSMFD, ASCE, Vol.89, SM1, February, pp. 1-31.

West, J.M. and Stuart, J.G., 'Oblique Loading Resulting from Interference Between Surface Footings on Sand', Proc. 6th International Conference on Soil Mechanics and Foundations.

T. William Lambe and Robert V. Whitman, Soil Mechanics, SI Version, Wiley Eastern Company Limited.